- Gauss-Legendre:

$$\int_{-1}^{1} f(x)dx \approx \sum_i w_i f(x_i) \quad \Rightarrow \quad \int_a^b g(y)dy \approx \sum_i \omega_i g(y_i)$$

- Gauss-Laguerre:

$$\int_0^{\infty} e^{-x} f(x)dx \approx \sum_i w_i f(x_i) \quad \Rightarrow \quad \int_a^{\infty} g(y)dy \approx \sum_i \omega_i g(y_i)$$

- Gauss-Hermite: Often used for integrating over Normal (μ, σ^2) expectations

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx \approx \sum_i w_i f(x_i) \quad \Rightarrow \quad \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right] g(y)dy \approx \sum_i \omega_i g(y_i)$$

The idea of quadratures is to choose optimal weights and nodes (ω_i, y_i) to improve accuracy with as few points as possible. The weights and nodes (w_i, x_i) are fixed for the standard cases, which we can use simple change of variables to integrate any integral of the above forms (for $f(\cdot)$ that are $\mathcal{C}_1$). Standard weights and nodes for N points can be easily found online (of a reference textbook).

Gauss-Legendre For Gauss-Legendre, transform the weights and nodes (w_i, x_i) from the standard bounds of support $(-1, 1)$ as follows:

$$\int_a^b g(y)dy = \frac{b-a}{2} \cdot \int_{-1}^{1} \underbrace{g\left(\frac{b-a}{2} \cdot x + \frac{b+a}{2}\right)}_{\equiv f(x)} dx \approx \sum_i \left(\frac{b-a}{2} \cdot w_i\right) \cdot g(y_i)$$

so set the weights and nodes as

$$\omega_i = \frac{b-a}{2} \cdot w_i, \quad y_i = \frac{b-a}{2} \cdot x_i + \frac{b+a}{2}$$

Gauss-Laguerre

$$\int_a^{\infty} g(y)dy = \int_0^{\infty} \underbrace{g(x+a)}_{\equiv f(x)} dx = \int_0^{\infty} e^x e^{-x} f(x)dx \approx \sum_i w_i e^{x_i} f(x_i) = \sum_i w_i e^{y_i - a} g(y_i)$$

so set the weights and nodes as

$$\omega_i = w_i \exp(y_i - a), \quad y_i = x_i + a$$

Gauss-Hermite Clearly $y = \sqrt{2}\sigma x + \mu$, so

$$\omega_i = w_i / \sqrt{\pi}, \quad y_i = \sqrt{2}\sigma x_i + \mu$$