

Inequality and Investment Risk

DEEQA Quantitative Macro

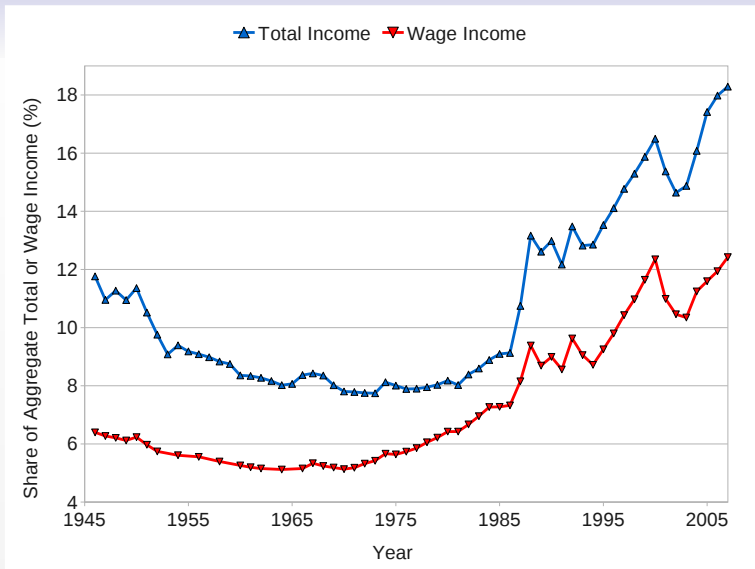
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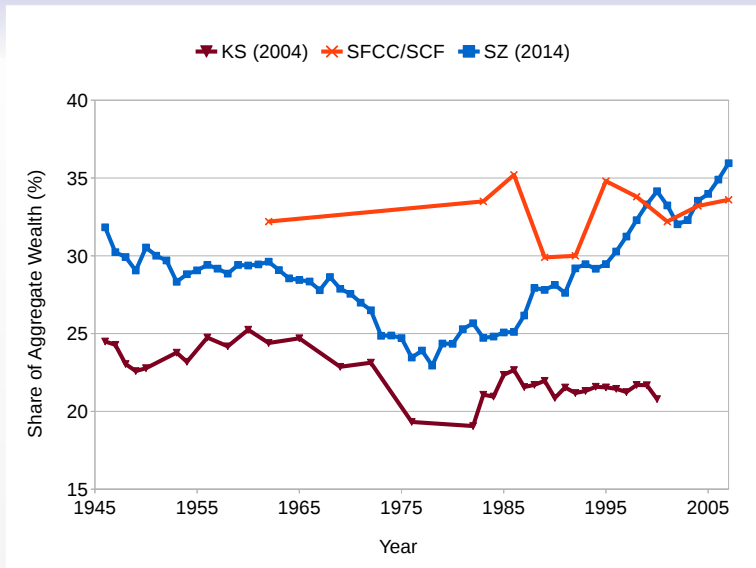
October 4, 2016

Game Plan for Today

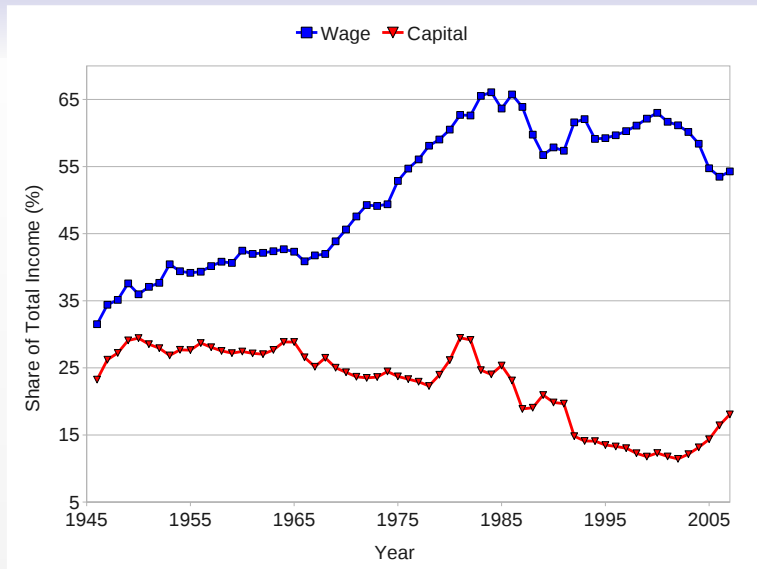
- Let's talk about inequality and its trends:
 1. Top income and wealth inequality
 2. Occupations and industries
- Focus on U.S. data (that's all I really know)



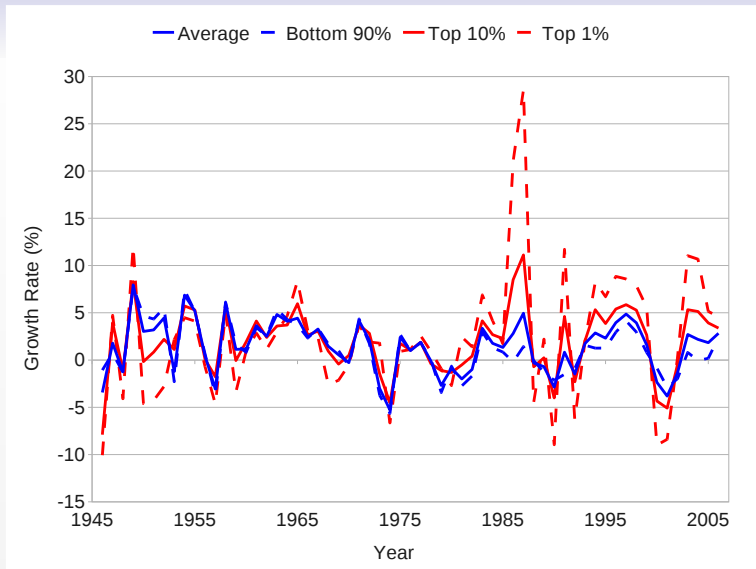
Top percentile shares of total and wage income (Piketty and Saez (2003)).



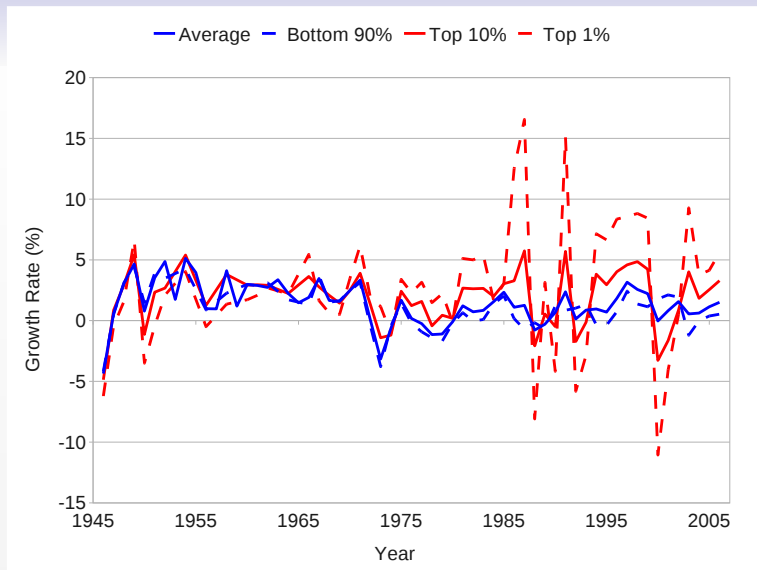
Top percentile wealth shares. Except for SCF, wealth shares are imputed and possibly inaccurate.



Top percentile composition of income (Piketty and Saez (2003)).



Total income growth rate volatility by fractile, 1946-2007 (Piketty and Saez (2003); Parker and Vissing-Jorgensen (2010)).

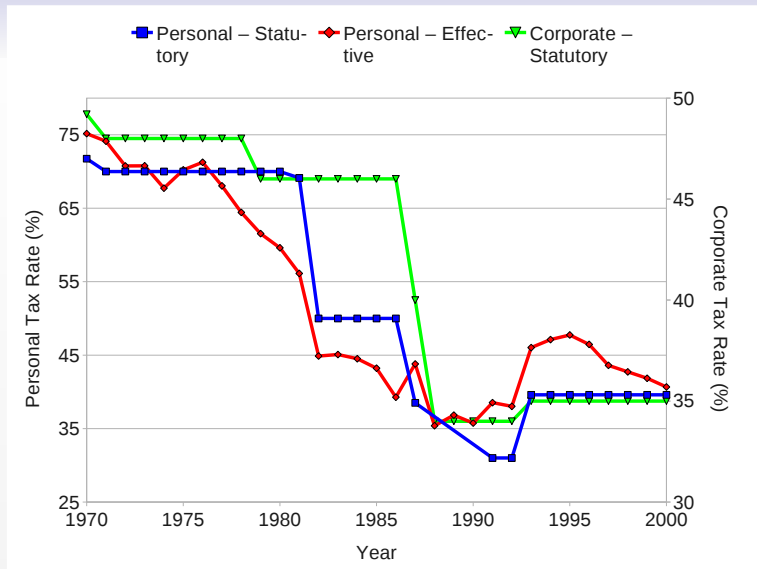


Wage income growth rate volatility by fractile, 1946-2007 (Piketty and Saez (2003); Parker and Vissing-Jorgensen (2010)).

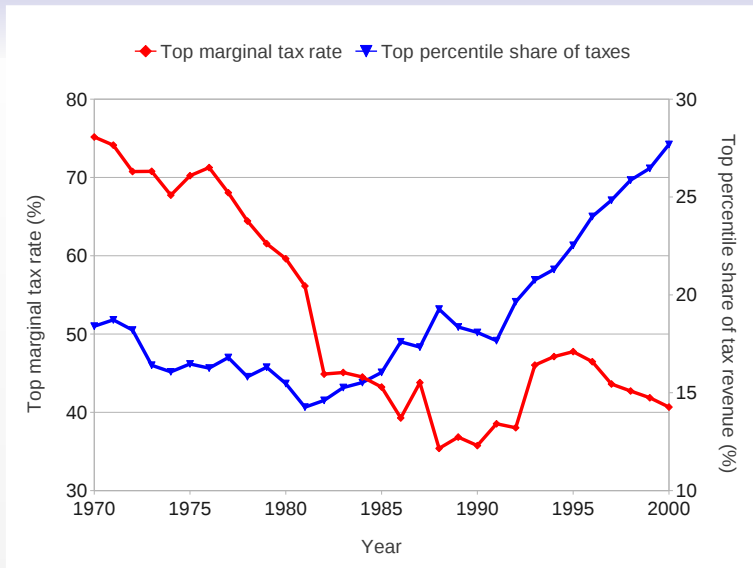
Exposure of Income Growth

	Year	T10%	T1%	T0.1%	T0.01%
Total Income	1946-1979	0.910	0.978	0.833	0.264
	1980-2007	1.481	2.808	4.076	4.671
Wage Income	1946-1979	0.857	0.646	0.352	0.144
	1980-2007	1.886	4.268	8.118	12.729

- OLS: group growth rate on population average growth rate (Piketty and Saez (2003); Parker and Vissing-Jorgensen (2010)).
- Guvenen et al. (2014, 2015) more detailed evidence on persistence and volatility of income shocks at the top (by sector, by gender, etc.)



Top marginal tax rates, 1970-2000 (IRS, Piketty and Saez (2007)).

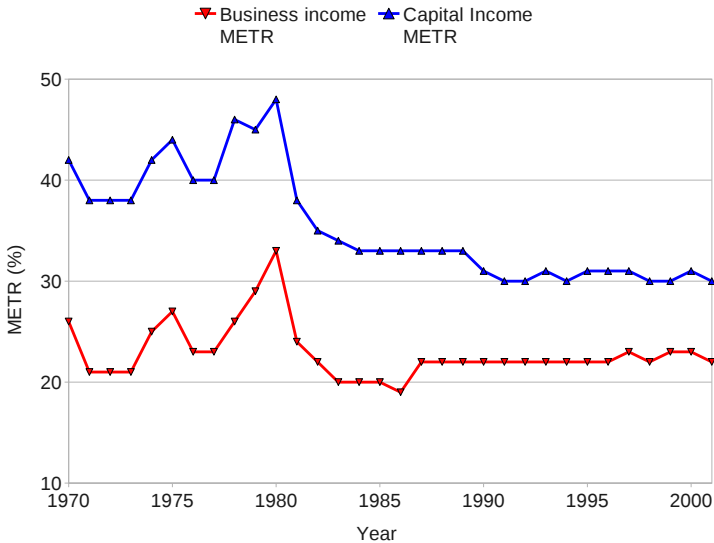


Top effective tax rate and top percentile share of total tax revenue, 1966-2000 (Piketty and Saez (2007)).

Some More Details

- Before we go theoretical, more details on U.S. tax structure
- Lots of low-hanging fruit here for future research
- Taxation becomes related to Public Economics
- * *(we're not really going to do anything about it though)*

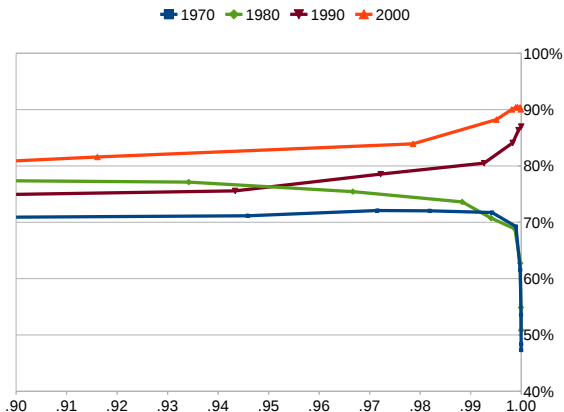
Effective Tax Rates on Capital



Business/Capital income METRs (Gravelle, 2007).

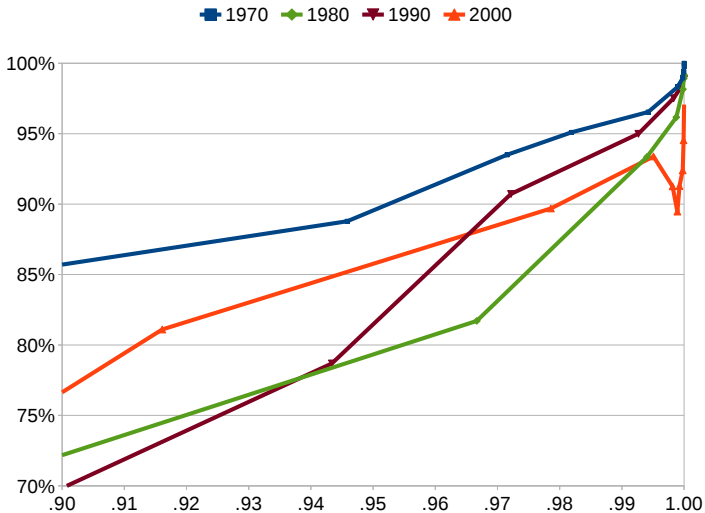
Much of income is *not* taxable, but less than before

Taxable / Broad Income:



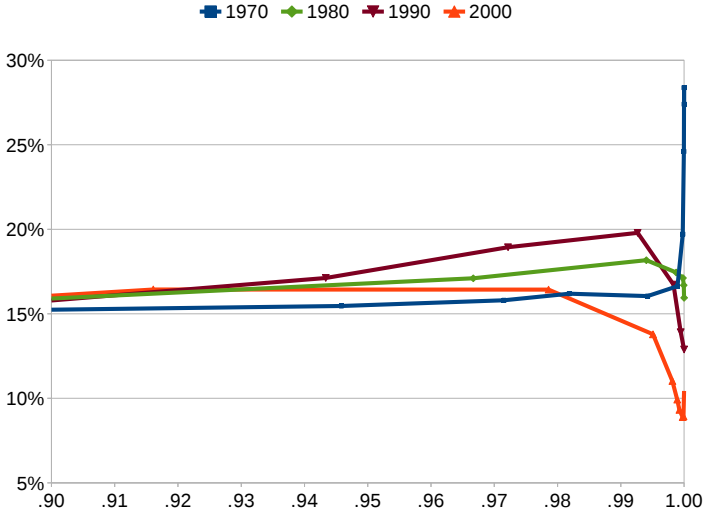
1. High income most sensitive to tax policy changes (Gruber and Saez, 2002)
2. But effect on executive managers transitory (Goolsbee, 2000)

Fraction of Itemizers by Percentile



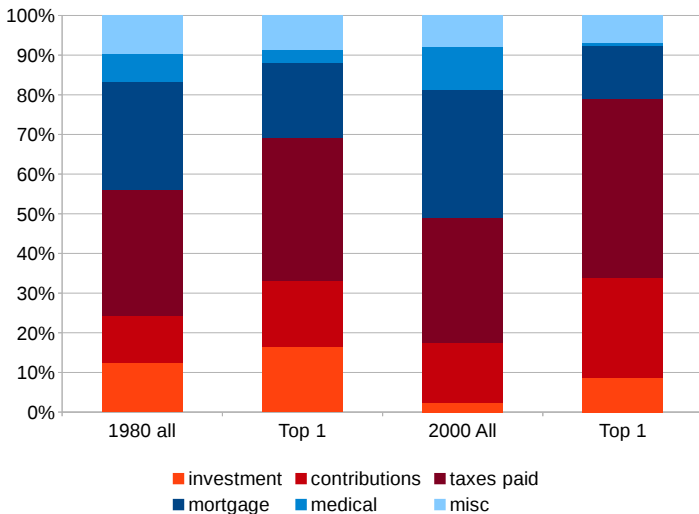
Almost all top income groups itemize, but less than before

Itemized Amounts by Percentile



Itemized amount is large, but less than before

Itemized Incomes of Top Percentile



Investment and property taxes; recently contributions deductions

How to Explain this?

1. Standard Huggett (1993); Aiyagari (1994) models with AR(1) earnings shocks cannot explain this (“symmetric” precautionary savings is not enough)
 2. Earnings risk?
 - Castañeda et al. (2003): “superstar” shocks
 - Model “managers” to create special high earnings groups
 3. Investment risk?
 - Angeletos (2007): everyone is a business owner with investment rate risk
 - Benhabib et al. (2014): risky asset in continuous time
 - Model “entrepreneurs” with collateral constraints: creates disproportionately high wealth concentration
- * Managers/entrepreneurs allows us to think about misallocation and (Roy (1951)) sorting

Investment Risk Model

- We solve a simpler variant of Angeletos (2007)
- Steady state; labor still inelastic. But everyone runs a business, hit by shocks $z_i \in [0, \infty)$

$$V(b, k, z_i) = \max_{c, b', k'} \{u(c) + \beta \mathbb{E} [V(b', k', z'_i) | z_i]\}$$

$$\text{s.t. } c + b' + k' = w + (1 + r)b + \max_n \{f_i(z_i, k, n) - wn\}$$

$$f(z_i, k, n) = (z_i k)^\alpha n^{1-\alpha} + (1 - \delta)(z_i k)$$

$$b' \geq -w/r, \quad k' \geq 0$$

Note that

- Business capital is fixed within-period (it's a state)
- Shock hits both output and *depreciated capital*
- NBL: use *deterministic* wage to pay interest on debt forever

Equivalence with Portfolio Choice Model

- Clearly, $n^* = [(1 - \alpha)/w]^{1/\alpha} z_i k$
- Plugging this in, we get an (almost) standard budget

$$c + b' + k' = w + (1 + r)b + (1 + R)z_i k$$

where $R \equiv \alpha [(1 - \alpha)/w]^{\frac{1-\alpha}{\alpha}} - \delta$.

- Capital is just risky asset you cannot short
- To incorporate borrowing constraint, define

$$a \equiv w/r + b$$

$$W_i = (1 + r)a + (1 + R)z_i k.$$

Linearity with Homotheticity

- Problem becomes

$$V(W_i, z_i) = \max_{a', k'} \left\{ u(W_i - a' - k') + \beta \mathbb{E} [V((1+r)a' + (1+R)z'_i k') | z_i] \right\}$$

- Optimality condition gives

$$\begin{aligned} & u'(W_i - a' - k') \\ &= \beta(1+r) \mathbb{E} [V'((1+r)a' + (1+R)z'_i k') | z_i] \\ &= \beta(1+R) \mathbb{E} [z'_i \cdot V'((1+r)a' + (1+R)z'_i k') | z_i] \end{aligned}$$

Solution

- Assume $u(c) = c^{1-\gamma}/(1-\gamma)$. Well known since Merton (1969, 1971, 1973) that policy rules are linear with homotheticity. So guess

$$c^*(W, z_i) = (1 - S_z)W,$$

$$a^*(W, z_i) = (1 - K_z)S_z W, \quad k^*(W, z_i) = K_z S_z W.$$

- Then $K_{z'}$ satisfies

$$\mathbb{E} \left[\left\{ 1 + r + K_{z'} \left[(1 + R)z' - (1 + r) \right] \right\}^{-\gamma} \right. \\ \left. \times \left[(1 + R)z' - (1 + r) \right] | z \right] = 0$$

Solution

- And the Euler equation implies that

$$(1 - S_z)[\beta(1 + r)]^{1/\gamma} / S_z =$$

$$\mathbb{E} \left[\left\{ (1 - S_{z'}) (1 + r + K_z [(1 + R)z' - (1 + r)]) \right\}^{-\gamma} | z \right]^{-1/\gamma}$$

- Given K_z , can solve for S_z .
- If no safe asset or z' independent of z , can get analytically closed form solutions

Solution

- We can approximate that

$$K_z \approx \mathbb{E} \left[[\log(1 + R) \log z' - \log(1 + r)] | z \right] / \gamma \sigma_z^2,$$

- Share of wealth held in risky asset is simple function of excess return and variance

$$\sigma^2 = \mathbb{V} [(1 + R)z' | z]$$

- Angeletos (2007) then derives the result that incomplete markets displays *lower* capital than in Aiyagari (1994)

Summary

- Angeletos (2007) in fact just an application of Merton (1969) to Bewley model
- More elegant in continuous time, can approximate tail of wealth distribution as Pareto
- Let's go continuous time
- Need to know
 1. Hamilton-Jacobi-Bellman equation
 2. Poisson jump process
 3. Brownian Motion (Wiener process) / Ito Calculus
 4. Kolmogorov Forward Equation

- Aiyagari, S. Rao**, “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics*, August 1994, 109 (3), 659–684.
- Angeletos, George-Marios**, “Uninsured idiosyncratic investment risk and aggregate saving,” *Review of Economic Dynamics*, 2007, 10 (1), 1 – 30.
- Benhabib, Jess, Alberto Bisin, and Shenghao Zhu**, “The Wealth Distribution in Bewley Models with Investment Risk,” NBER Working Papers 20157, National Bureau of Economic Research, Inc May 2014.
- Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull**, “Accounting for the U.S. Earnings and Wealth Inequality,” *Journal of Political Economy*, 2003, 111, 818–857.
- Goolsbee, Austan**, “What Happens When You Tax the Rich? Evidence from Executive Compensation,” *Journal of Political Economy*, April 2000, 108 (2), 352–378.

Gravelle, Jane G., “Historical Effective Marginal Tax Rates on Capital Income,” Technical Report, Congressional Research Service Report for Congress 2007.

Gruber, Jon and Emmanuel Saez, “The elasticity of taxable income: evidence and implications,” *Journal of Public Economics*, April 2002, 84 (1), 1–32.

Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song, “What Do Data on Millions of U.S. Workers Reveal about Life-Cycle Earnings Risk?,” NBER Working Papers 20913, National Bureau of Economic Research, Inc January 2015.

—, **Greg Kaplan, and Jae Song**, “How Risky Are Recessions for Top Earners?,” *American Economic Review*, May 2014, 104 (5), 148–53.

Huggett, Mark, “The Risk Free Rate in Heterogeneous-Agent, Incomplete-Insurance Economies,” *Journal of Economic Dynamics and Control*, 1993, 17, 953–969.

- Merton, Robert C.**, “Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case,” *The Review of Economics and Statistics*, August 1969, 51 (3), 247–57.
- Merton, Robert C.**, “Optimum consumption and portfolio rules in a continuous-time model,” *Journal of Economic Theory*, December 1971, 3 (4), 373–413.
- Merton, Robert C.**, “An Intertemporal Capital Asset Pricing Model,” *Econometrica*, September 1973, 41 (5), 867–87.
- Parker, Jonathan A. and Annette Vissing-Jorgensen**, “The Increase in Income Cyclicalness of High-Income Households and Its Relation to the Rise in Top Income Shares,” *Brookings Papers on Economic Activity*, 2010, 41 (2 (Fall)), 1–70.
- Piketty, Thomas and Emmanuel Saez**, “Income Inequality in the United States, 1913-1998,” *Quarterly Journal of Economics*, 2003, 118 (1), 1–39.
- **and** —, “How Progressive is the U.S. Federal Tax System? A Historical and International Perspective,” *Journal of Economic Perspectives*, Winter 2007, 21 (1), 3–24.

Roy, A. D., “Some Thoughts on the Distribution of Earnings,”
Oxford Economic Papers, 1951, 3 (2), pp. 135–146.