Inequality and Investment Risk

DEEQA Quantitative Macro

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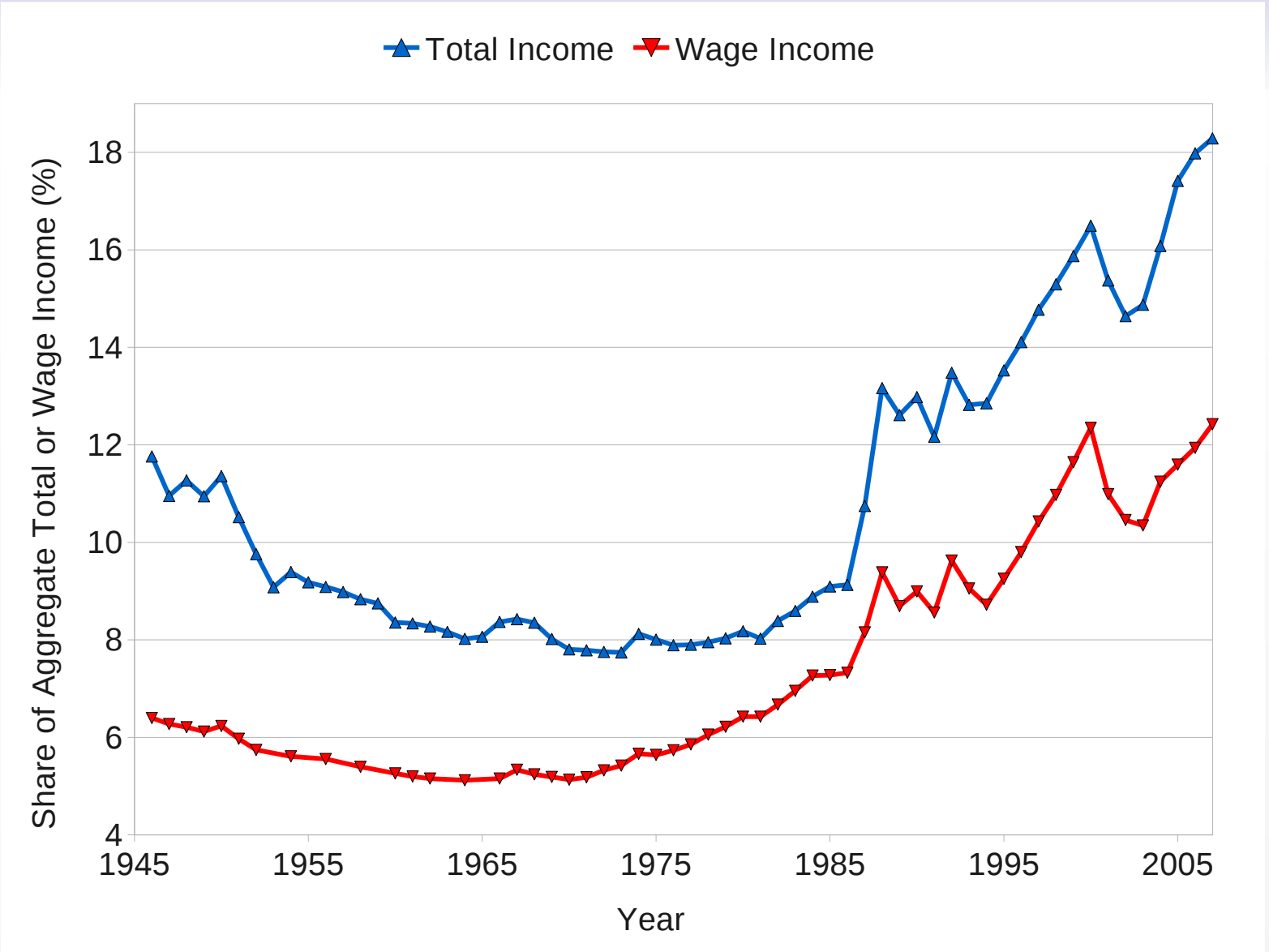
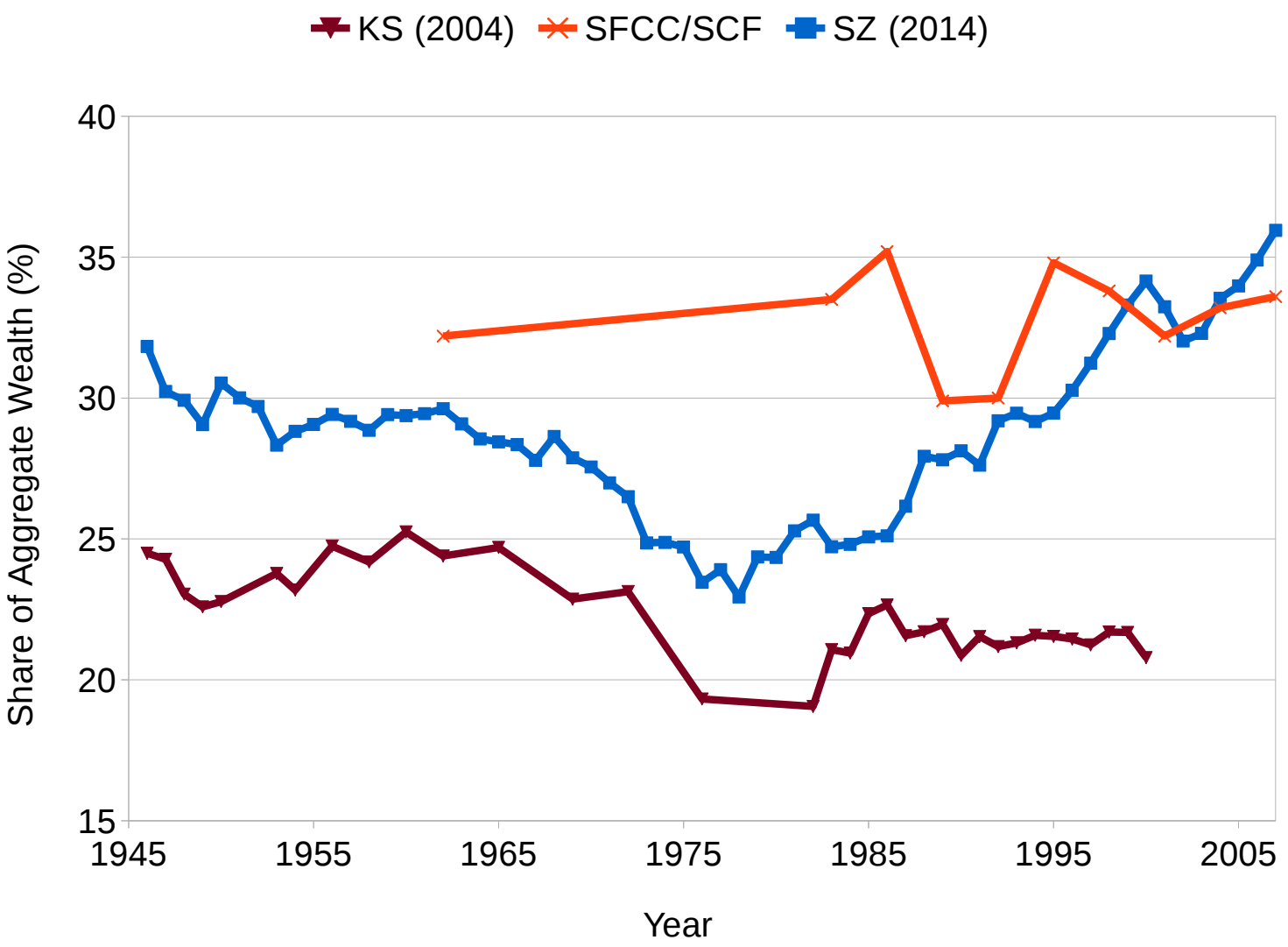
Game Plan for Today

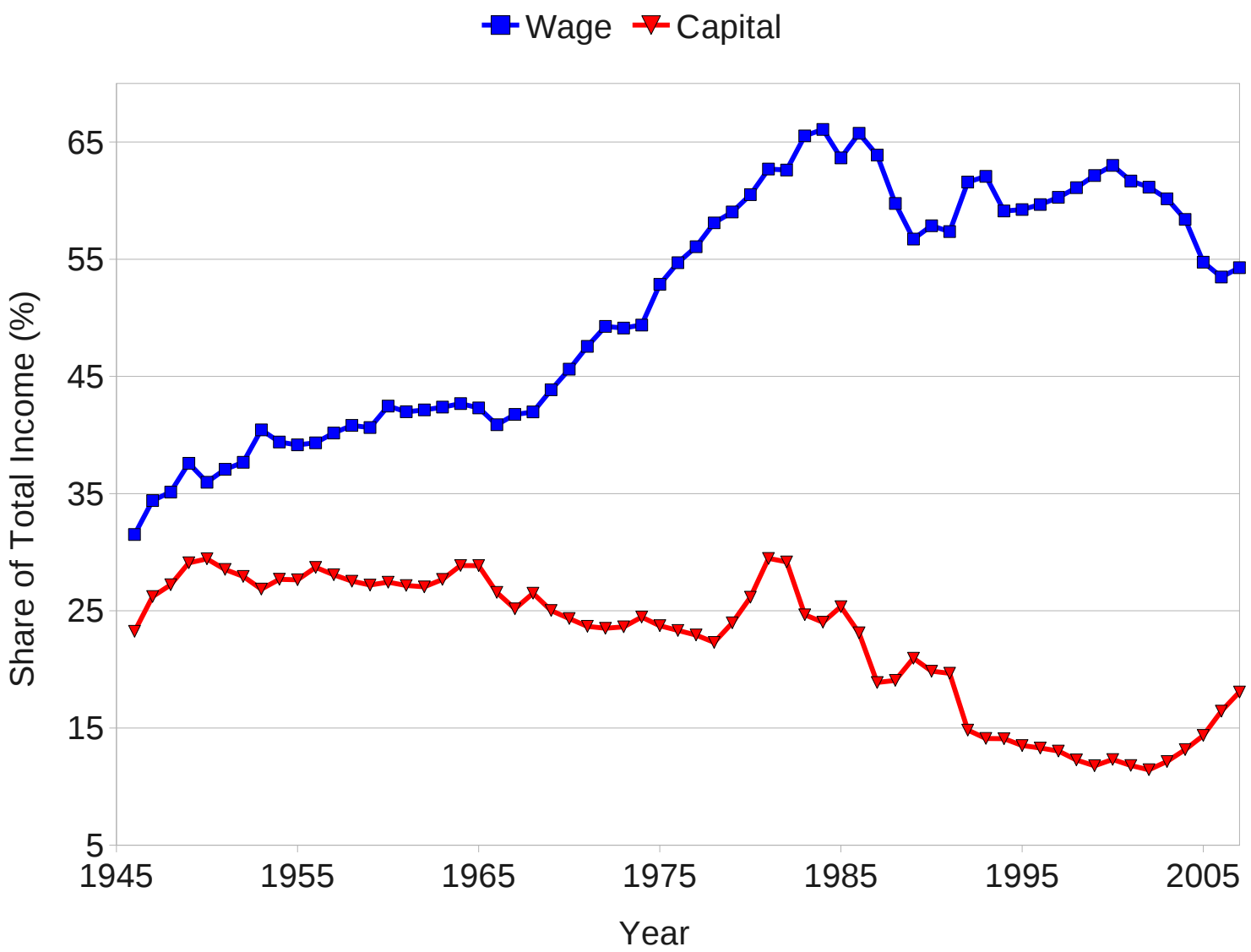
- Let's talk about inequality and its trends:
  1. Top income and wealth inequality
  2. Occupations and industries
- Focus on U.S. data (that's all I really know)



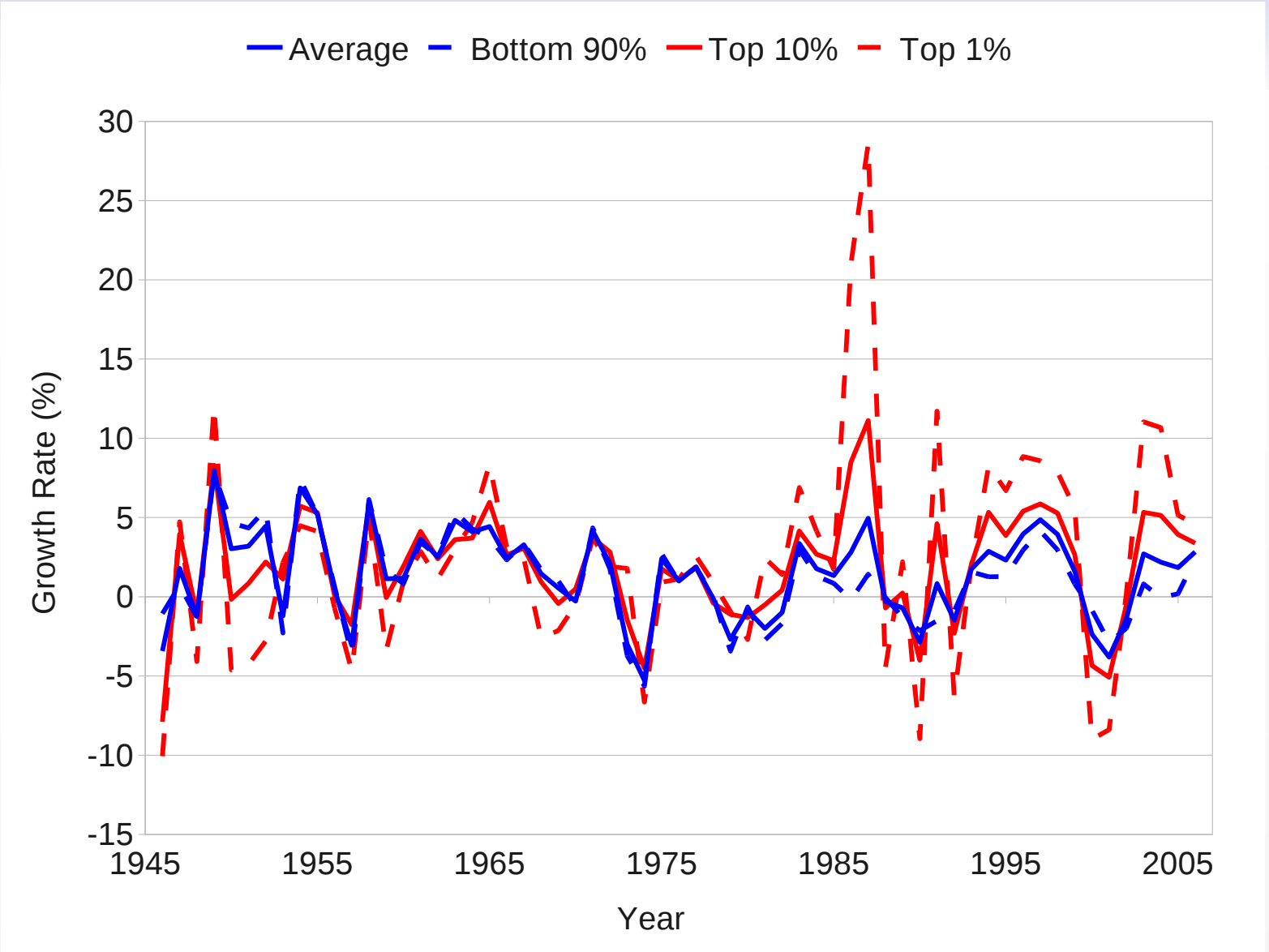
Top percentile shares of total and wage income (Piketty and Saez (2003)).



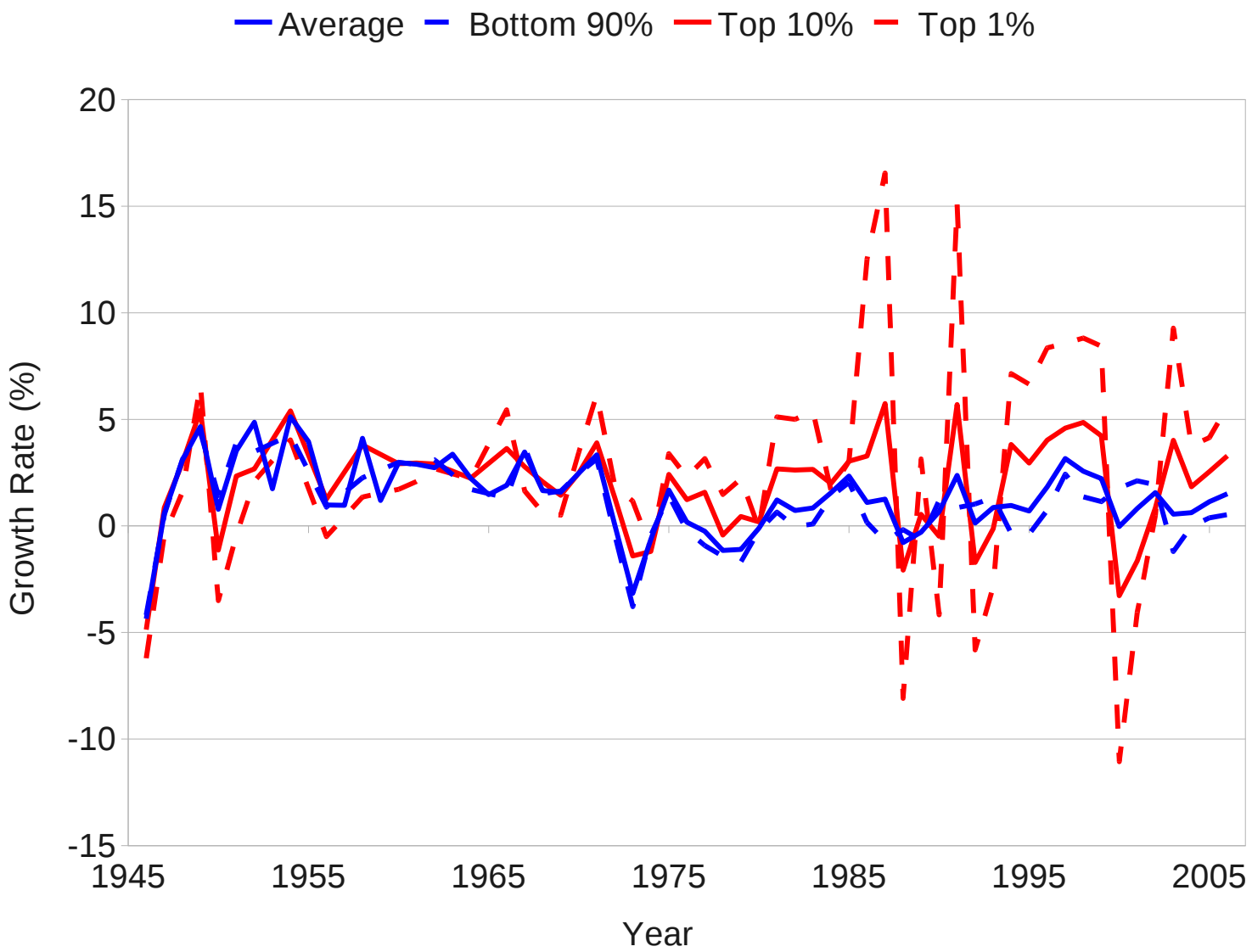
Top percentile wealth shares. Except for SCF, wealth shares are imputed and possibly inaccurate.



Top percentile composition of income (Piketty and Saez (2003)).



Total income growth rate volatility by fractile, 1946-2007 (Piketty and Saez (2003); Parker and Vissing-Jorgensen (2010)).

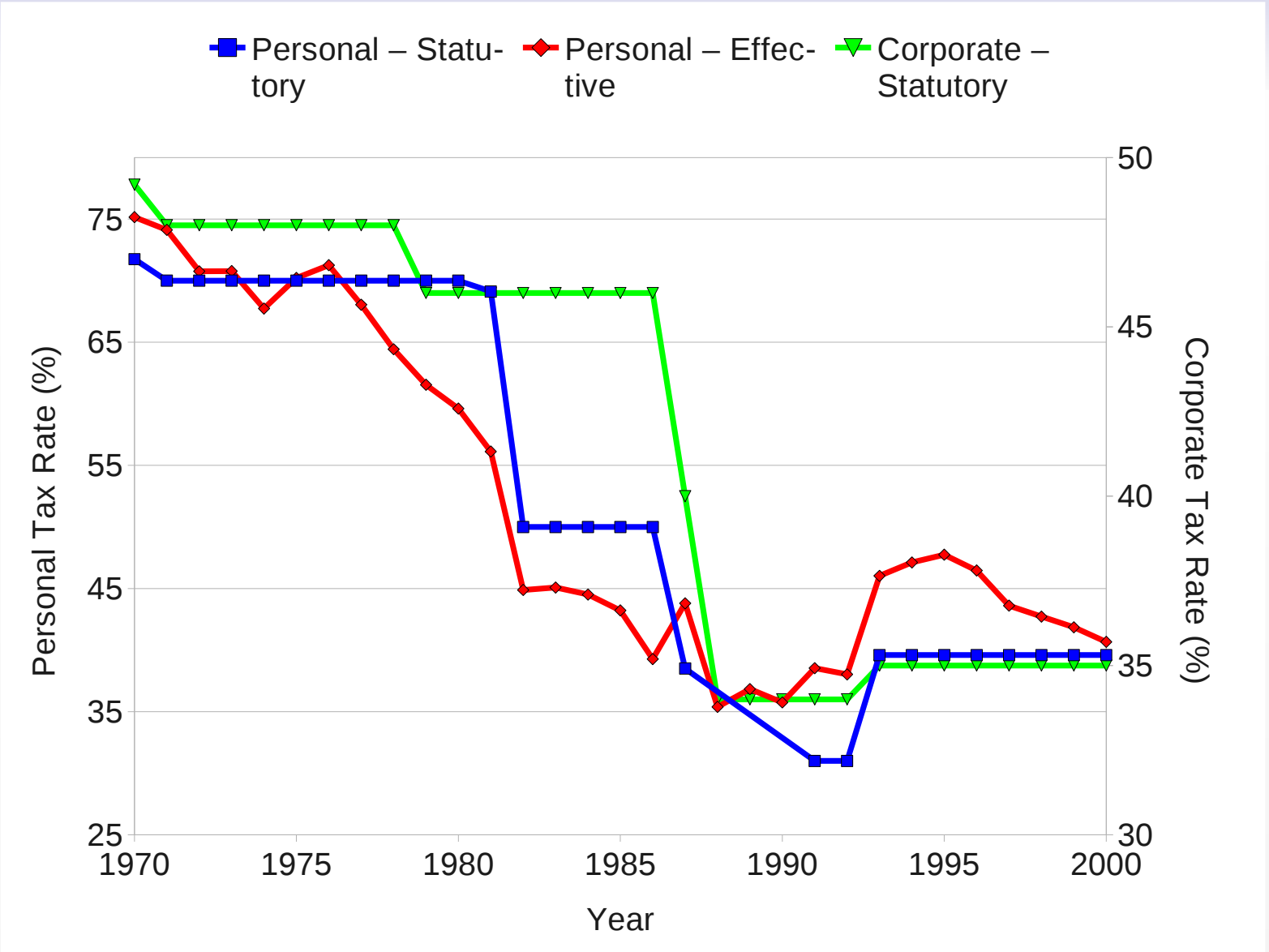


Wage income growth rate volatility by fractile, 1946-2007 (Piketty and Saez (2003); Parker and Vissing-Jorgensen (2010)).

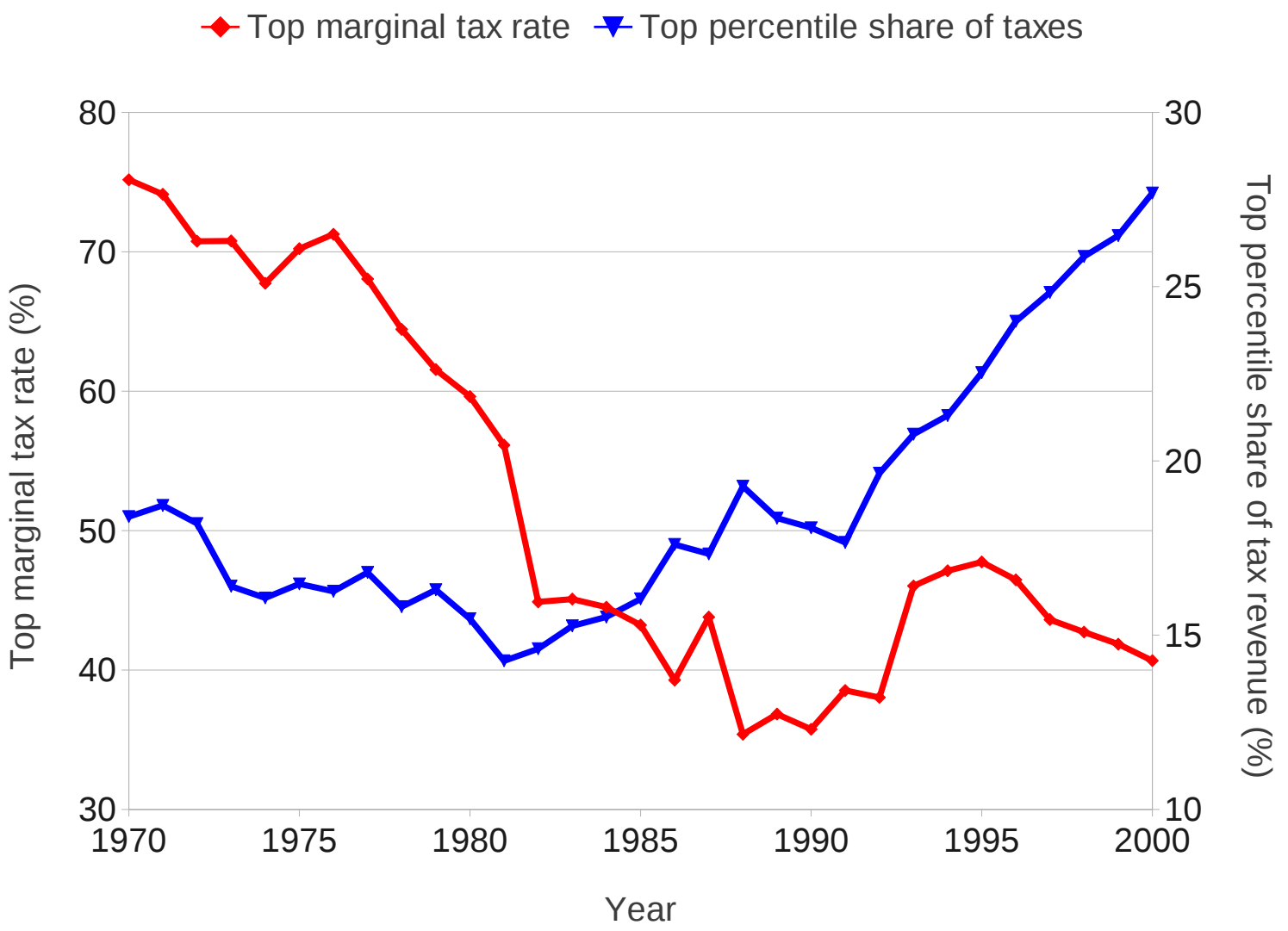
Exposure of Income Growth

	Year	T10%	T1%	T0.1%	T0.01%
Total Income	1946-1979	0.910	0.978	0.833	0.264
	1980-2007	1.481	2.808	4.076	4.671
Wage Income	1946-1979	0.857	0.646	0.352	0.144
	1980-2007	1.886	4.268	8.118	12.729

- OLS: group growth rate on population average growth rate (Piketty and Saez (2003); Parker and Vissing-Jorgensen (2010)).
- Guvenen et al. (2014, 2015) more detailed evidence on persistence and volatility of income shocks at the top (by sector, by gender, etc.)



Top marginal tax rates, 1970-2000 (IRS, Piketty and Saez (2007)).



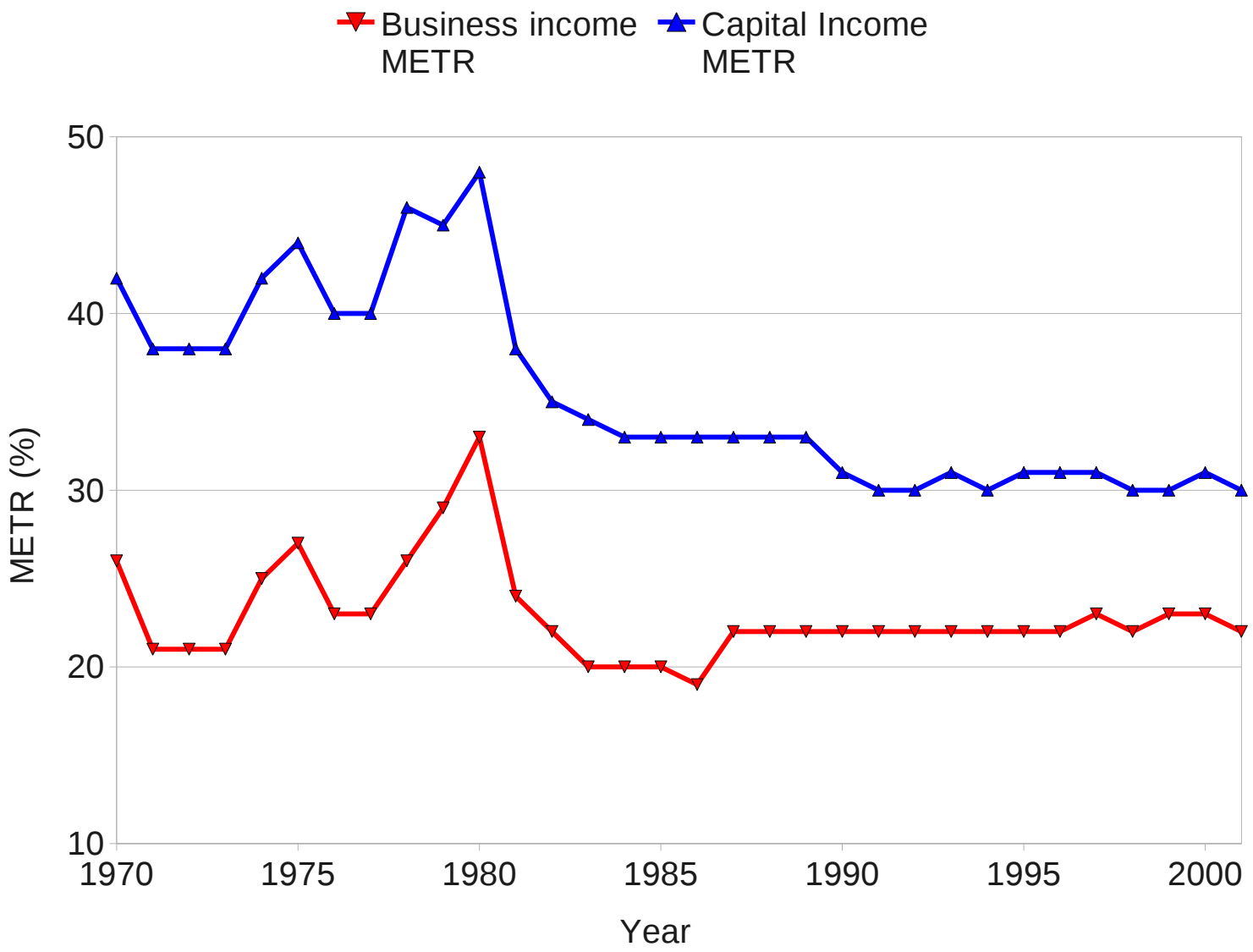
Top effective tax rate and top percentile share of total tax revenue, 1966-2000 (Piketty and Saez (2007)).

Some More Details

- Before we go theoretical, more details on U.S. tax structure
- Lots of low-hanging fruit here for future research
- Taxation becomes related to Public Economics

* *(we're not really going to do anything about it though)*

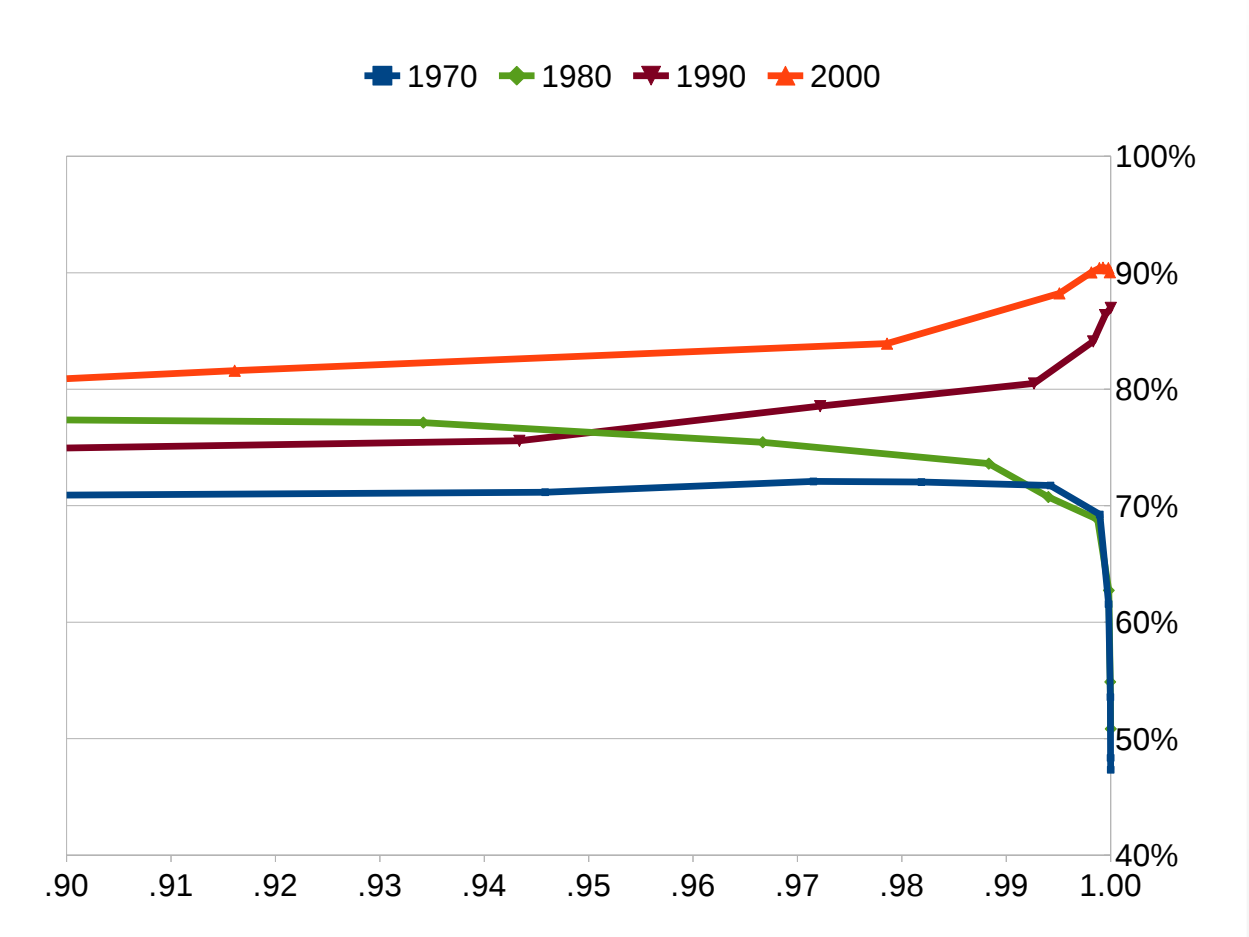
Effective Tax Rates on Capital



Business/Capital income METRs (Gravelle, 2007).

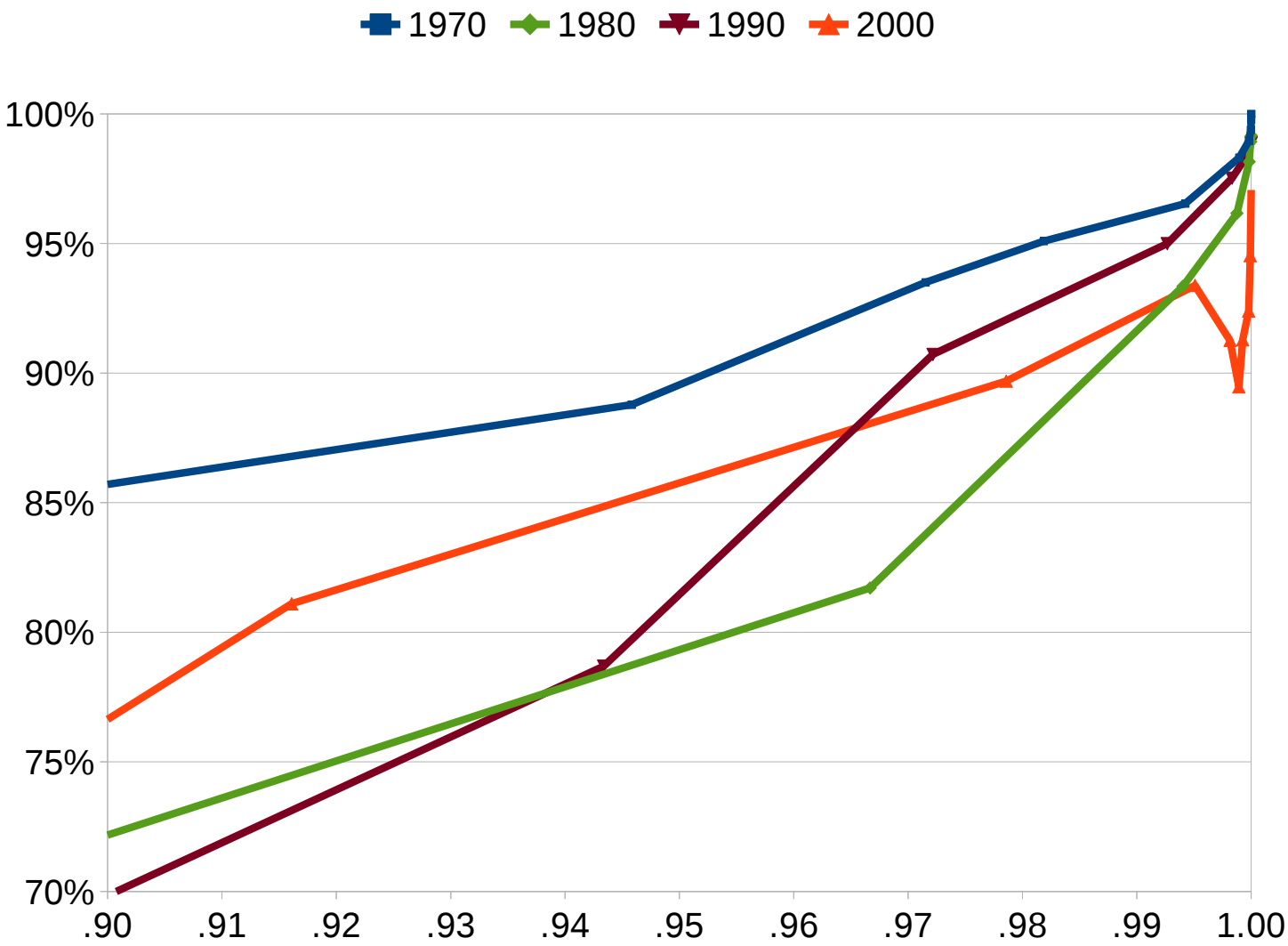
Much of income is *not* taxable, but less than before

Taxable / Broad Income:



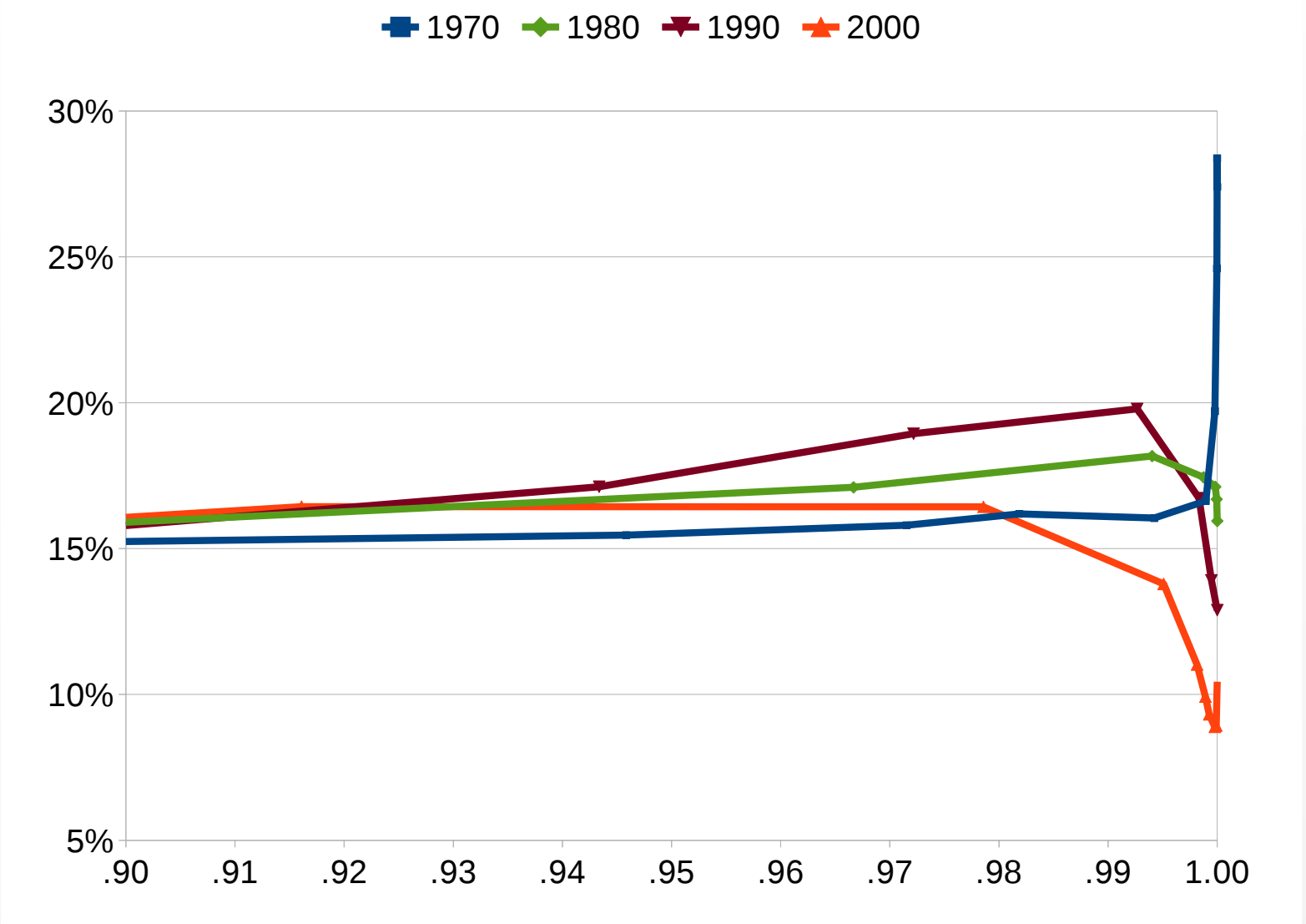
1. High income most sensitive to tax policy changes (Gruber and Saez, 2002)
2. But effect on executive managers transitory (Goolsbee, 2000)

Fraction of Itemizers by Percentile



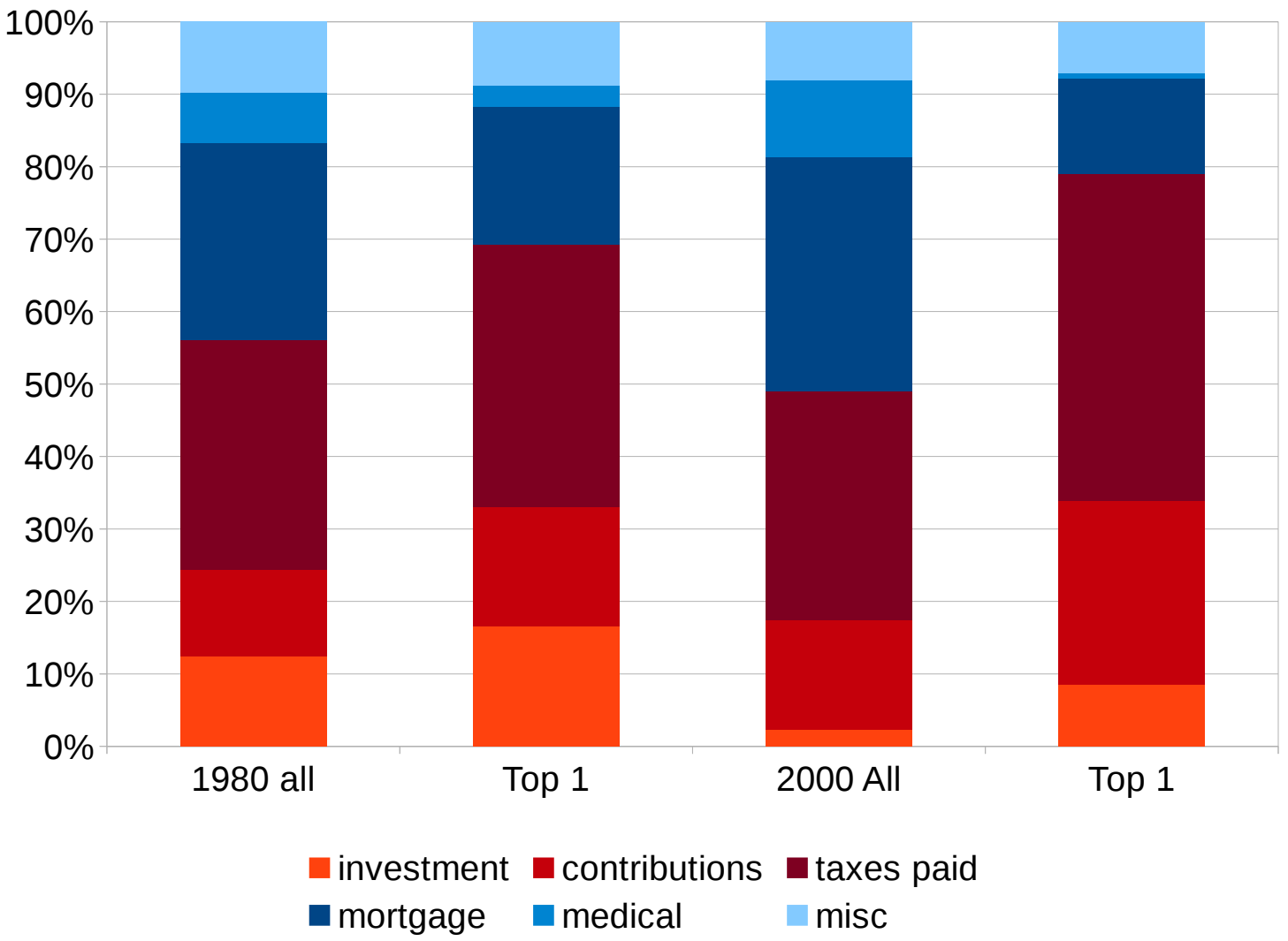
Almost all top income groups itemize, but less than before

Itemized Amounts by Percentile



Itemized amount is large, but less than before

Itemized Incomes of Top Percentile



Investment and property taxes; recently contributions deductions

How to Explain this?

1. Standard Huggett (1993); Aiyagari (1994) models with AR(1) earnings shocks cannot explain this (“symmetric" precautionary savings is not enough)
2. Earnings risk?
   - Castañeda et al. (2003): “superstar" shocks
   - Model “managers” to create special high earnings groups
3. Investment risk?
   - Angeletos (2007): everyone is a business owner with investment rate risk
   - Benhabib et al. (2014): risky asset in continuous time
   - Model “entrepreneurs" with collateral constraints: creates disproportionately high wealth concentration

* Managers/entrepreneurs allows us to think about misallocation and (Roy (1951)) sorting

Investment Risk Model

- We solve a simpler variant of Angeletos (2007)
- Steady state; labor still inelastic. But everyone runs a business, hit by shocks $z_i \in [0, \infty)$

$$V(b, k, z_i) = \max_{c, b', k'} \left\{ u(c) + \beta \mathbb{E}\left[V(b', k', z_i') | z_i \right] \right\}$$

$$\text{s.t.} \quad c + b' + k' = w + (1 + r)b + \max_{n} \left\{ f_i(z_i, k, n) - wn \right\}$$

$$f(z_i, k, n) = (z_i k)^{\alpha} n^{1-\alpha} + (1 - \delta)(z_i k)$$

$$b' \geq -w/r, \quad k' \geq 0$$

Note that

- Business capital is fixed within-period (it's a state)
- Shock hits both output and *depreciated capital*
- NBL: use *deterministic* wage to pay interest on debt forever

Equivalence with Portfolio Choice Model

- Clearly, $n^* = [(1-\alpha)/w]^{1/\alpha} z_i k$
- Plugging this in, we get an (almost) standard budget

$$c + b' + k' = w + (1+r)b + (1+R)z_i k$$

where $R \equiv \alpha [(1-\alpha)/w]^{\frac{1-\alpha}{\alpha}} - \delta$.

- Capital is just risky asset you cannot short
- To incorporate borrowing constraint, define

$$a \equiv w/r + b$$
$$W_i = (1+r)a + (1+R)z_i k.$$

Linearity with Homotheticity

- Problem becomes

$$V(W_i, z_i) = \max_{a', k'} \Big\{ u(W_i - a' - k') + \beta \mathbb{E}\left[V\left((1+r)a' + (1+R)z_i' k' \right) | z_i \right] \Big\}$$

- Optimality condition gives

$$\begin{aligned} & u'(W_i - a' - k') \\ =& \beta(1+r)\mathbb{E}\left[V'\left((1+r)a' + (1+R)z_i' k' \right) | z_i \right] \\ =& \beta(1+R)\mathbb{E}\left[z_i' \cdot V'\left((1+r)a' + \left(1+R\right)z_i' k' \right) \right) | z_i \right] \end{aligned}$$

Solution

- Assume $u(c) = c^{1-\gamma}/(1-\gamma)$. Well known since Merton (1969, 1971, 1973) that policy rules are linear with homotheticity. So guess

$$c^*(W, z_i) = (1 - S_z)W,$$
$$a^*(W, z_i) = (1 - K_z)S_z W, \quad k^*(W, z_i) = K_z S_z W.$$

- Then $K_{z'}$ satisfies

$$\mathbb{E}\Big[\left\{1 + r + K_{z'} \left[(1+R)z' - (1+r)\right]\right\}^{-\gamma} \times \left[(1+R)z' - (1+r)\right] | z \Big] = 0$$

Solution

- And the Euler equation implies that

$$(1 - S_z)[\beta(1 + r)]^{1/\gamma}/S_z =$$

$$\mathbb{E}\Big[\left\{(1 - S_{z'})(1 + r + K_z[(1 + R)z' - (1 + r)])\right\}^{-\gamma} |z\Big]^{-1/\gamma}$$

- Given K_z, can solve for S_z.
- If no safe asset or z' independent of z, can get analytically closed form solutions

Solution

- Denote the variance of $\log z$ as

$$\sigma^2 = \mathbb{V}\left[\log z' | z\right]$$

- and let ρ denote the risk-adjusted return:

$$\begin{aligned}\rho \approx & (1 - K_z) \log r + K_z \left(\log R + \mathbb{E}\left[\log z' | z\right]\right) \\ & + \frac{1}{2}\sigma^2 K_z \left[(1 - K_z) + (1 - \gamma) K_z\right]\end{aligned}$$

Solution

- We can approximate that

$$K_z \approx \frac{\mathbb{E}\left[\log z' | z\right] + \sigma^2/2}{\gamma \sigma^2},$$

- Share of wealth held in risky asset is simple function of excess return and variance
- Angeletos (2007) then derives the result that incomplete markets displays *lower* capital than in Aiyagari (1994)

Summary

- Angeletos (2007) in fact just an application of Merton (1969) to Bewley model
- More elegant in continuous time, can approximate tail of wealth distribution as Pareto
- Let's go continuous time
- Need to know
  1. Hamilton-Jacobi-Bellman equation
  2. Poisson jump process
  3. Brownian Motion (Wiener process) / Ito Calculus
  4. Kolmogorov Forward Equation

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