Inequality, Occupational Choice, and Collateral Constraints

Ph.D. Quantitative Macro

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Top Income Inequality

1. Huggett (1993); Aiyagari (1994): assets have upperbound, i.e. rich save less. Precautionary motive declines with DARA
2. Castañeda et al. (2003) reverses this with infrequent “superstar" shocks that make rich save more in fear of becoming poor
3. Angeletos (2007); Benhabib et al. (2014): investment risk makes rich save proportionately more
4. Occupation choice with collateral constraints: fear of facing constraint make individuals save more in anticipation of becoming rich

$\Rightarrow$ Begin to think about misallocation and sorting

Occupational Choice in General Eqm

- Let's start with Lucas (1978) "span-of-control," static model
- Originally proposed to study firm size distribution, but often exploited to analyze entrepreneurial activity
- In dynamic, quantitative versions with constraints:
  - Quadrini (2000): income class mobility
  - Cagetti and De Nardi (2006): wealth inequality
  - Buera et al. (2011): financial frictions and development

Key elements

1. Ability: the main element in the Lucas (1978) model
2. Collateral constraints: in dynamics, *wealth accumulation* induced by the constraint becomes an important state in addition to ability
3. Many studies use an *endogenous* borrowing constraint

c.f. Does it matter whether the constraint is en- or exogenous?

Topics

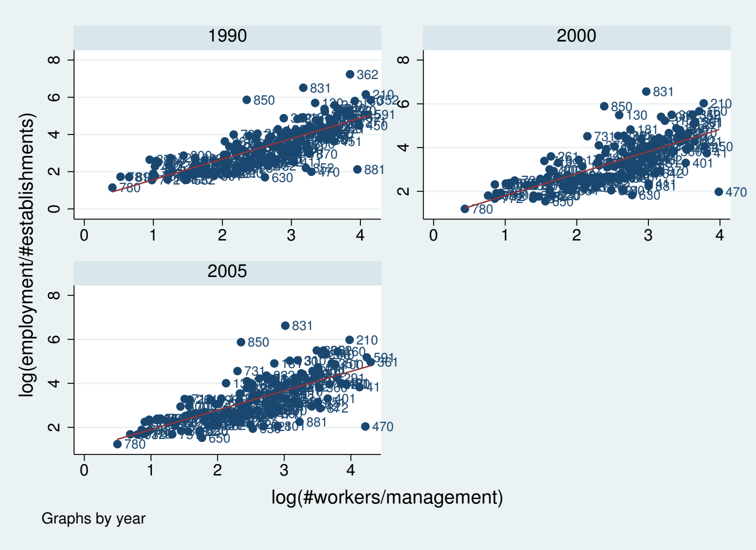
We will study these mechanisms one by one:

1. Original Lucas model - size of firm determined by entrepreneurial ability
2. Individual entrerpeneur's savings behavior and its implications for wealth inequality and capital misallocation (Buera (2009); Moll (2014))
3. Endogneous borrowing constraints and variants (Buera and Shin (2013)); entry distortions (Buera et al. (2011); Midrigan and Xu (2013))
4. Misallocation and TFP differences (Restuccia and Rogerson (2008); Hsieh and Klenow (2009))

Lucas' Span of Control Model (modified)

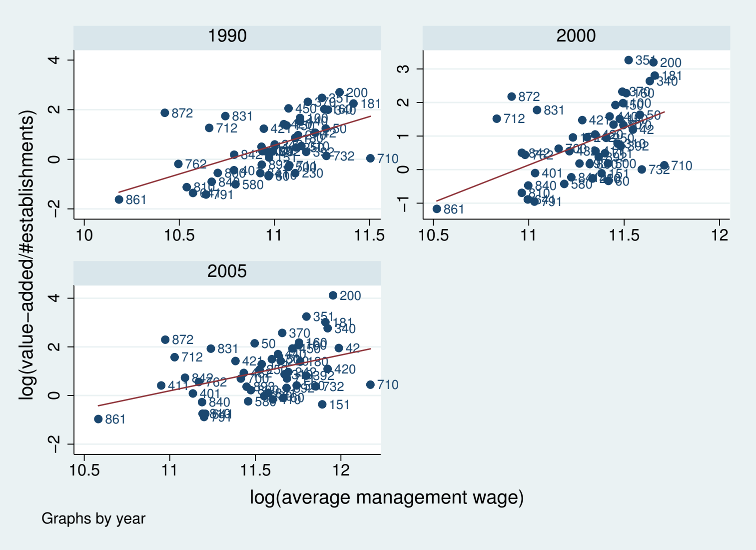
- Original model is static, but let's study a dynamic one
- With perfect credit markets (subject only to a natural borrowing limit), it doesn't matter
- Recent models add collateral constraints; drops perfect credit assumption
- Empirical
    1. establishment size correlates with workers per management
    2. firm size correlations with CEO compensation (Gabaix and Landier, 2008)

Establishment Size and Worker-Mgr Ratios



By SIC industry code: SUSB and Census/ACS

Establishment Output and Mgr Wages



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Environment

1. Technology: DRS in factor inputs

$$f(e_i, k, l) = e_i^{\nu}(k^{\alpha} l^{1-\alpha})^{1-\nu}$$

   Preferences:

$$\int_0^{\infty} e_i^{-\rho t} \frac{c_i(t)^{1-\sigma}}{1-\sigma} dt$$

   Abilities:

$$e_i \sim G(e_i), \quad \text{non-degenerate}$$

2. Entrepreneurs run firms and hire workers. In equilibrium all markets (in particular the labor market) must clear.
3. No frictions: equilibrium equal to planner's solution

Environment

With perfect credit assumption, can show that the equilibrium is characterized by a cutoff e^*:

1. For the planner's problem, one can show that otherwise, there exists a Pareto improvement, a contradiction
2. For the competitive equilibrium, one can show that otherwise, there exists a profitable exchange among agents, a contradiction

Aggregate Dynamics and Endogenous TFP

- Can write the aggregate production function as

$$F(K_t) = \underbrace{[1 - G(e^*)]}_{\text{extensive}} \underbrace{\mathbb{E}[e^\nu | e > e^*]}_{\text{intensive}} \Big[K_t^\alpha \underbrace{L_t}_{G(e^*)}{}^{1-\alpha}\Big]^{1-\nu}$$

and aggregate capital evolves as

$$\dot{K}(t) = F(K_t) - \delta K_t - C_t.$$

- So, Lucas model is a model of endogenous TFP
- Depends on mass (extensive margin) and average manager ability (intensive margin)
- In stochastic case, the more sorting, the higher the TFP (since we don't get perfect sorting with stochasticity)

Some Questions

- Initial wealth distribution does not matter for aggregate dynamics - why?
- We do not need to consider the actual occupations of the individuals in this case - why?
- Speed of convergence is slow in this economy - why?

* Can solve the planner's Hamiltonian and linearize around the steady state for exact speed

Span-of-Control and Capital Misallocation

Here's an exercise:

- Now suppose there are *no* credit markets altogether.
- Assume there are only two levels of ability, $\{e_L, e_H\}$
- Assume all high ability agents become entrepreneurs.
- Assume only high ability agents hold initial capital, at equal amounts. Derive the entrepreneur's factor demands (k_t, l_t) taking (r_t, w_t) as given.
- Express entrepreneurial profits as a function of (k_t, K_t).
- Derive the equilibrium Hamiltonian dynamics.
- Linearize around the steady state. What's the speed of convergence?
- What happens if all the capital were held by the low ability agents instead?

Intuition for Exercise

- Wealth inequality higher in steady state
- Speed of convergence is faster
- This is because high ability agents save up so that they can self-finance
- Let's think about this more formally

Individual Entrepreneur's Problem

Buera (2009):

- No equilibrium, studies savings behavior of individual entrepreneurs
- Ability is fixed for each individual
- Depending on abilities and initial asset levels, long run assets and savings behavior will differ
- Provides clear intuition for the underlying individual mechanism in more complex entrepreneurial models

Individual Entrepreneur's Problem

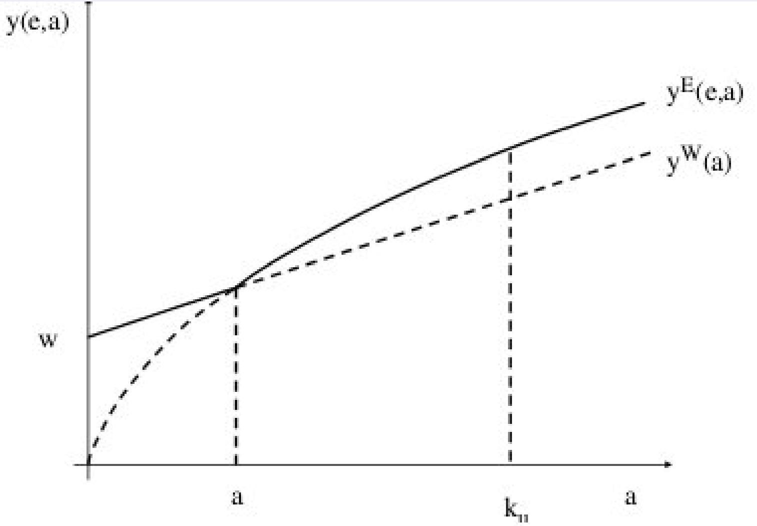
$$\max_{c(t),a(t),k(t)} \int_0^\infty e^{-\rho t} u(c(t))dt$$

s.t.

$$\begin{aligned}
\dot{a}(t) &= y(e, a(t)) - c(t) \\
y(e, a(t)) &= \max\{y^E(e, a(t)), y^W(e, a(t))\} \\
y^E(e, a(t)) &= \max_{k \leq a}\{f(e, k) + r(a(t) - k)\} \\
y^W(e, a(t)) &= w + ra(t)
\end{aligned}$$

Define indifference state $\overline{a}(e)$ and unconstrained optimum $k^*(e)$:

$$\begin{aligned}
f(e, \overline{a}(e)) &= w + r\overline{a}(e) \\
k^*(e) &= \arg\max_k\{f(e, k) - rk\}
\end{aligned}$$



Occupation-dependent incomes (Buera, 2009).

Long-run Convergence

In long run, agents converge to $[a(e, a_0), c(e, a_0)]$: assets and incomes dependent on their ability and initial wealth levels, which are one of the two

- Hand-to-Mouth (HtM) state: $(0, w)$, or
- Entrepreneurial state: $(a^*(e), c^*(e))$.

Proposition

There exists cutoffs $0 < \underline{e} < \overline{e} < \infty$ *s.t.*

- *Low ability agents (*$e \leq \underline{e}$*) converge to* $(0, w)$ *for all* a_0*.*
- *For mid ability agents (*$e \in (\underline{e}, \overline{e})$*), there exists an ability dependent cutoff* $\underline{a}(e)$ *s.t. agents with* $a_0 = \underline{a}(e)$ *are indifferent between converging to either state. If* $a_0 < \underline{a}(e)$*, the HtM is preferred, and vice versa.*

Long-run Implications

Corollary

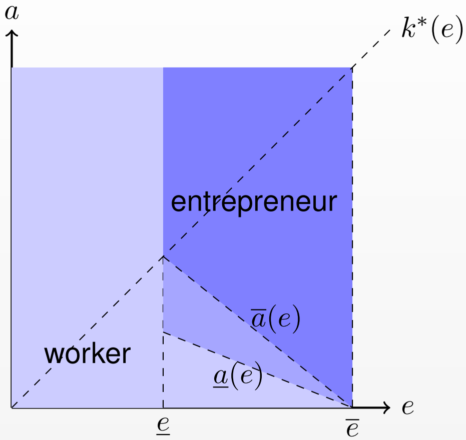
- *Savings rate of those who converge to the entrepreneurial state is higher than otherwise.*
- *For these guys, growth rate of consumption increases once they become entrepreneurs (save less).*

Proposition

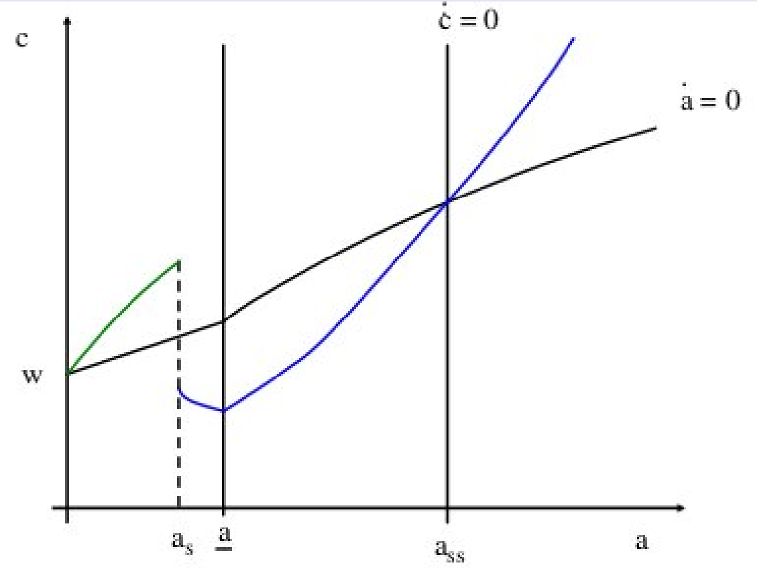
If $\underline{a}(e) < \overline{a}(e)$, $\underline{a}'(e) < 0$. *For* $e \approx \underline{e}$, $\underline{a}$ *increases with the interest rate. For* $e \approx \overline{e}$, $\underline{a}$ *decreases with the interest rate.*

Substitution effect dominates income effect for higher e: future income is higher

Long Run Occupational Choice



Given an exogenous distribution for e, the steady state wealth distribution will have a large mass at 0, with extremely wealthy agents with $e > \underline{e}$ at $a^*(e)$.



Long-run dynamics (Buera (2009)).

Implications for Long Run Wealth Holdings

- Moll (2014): stochastic version with collateral constraint λ
- Tricks for a closed form solution of SS wealth distribution:
    - Entrepreneurs hire workers according to $f(e, k, l)$, which is HD1 in (k, l)
    - Assume $\lambda = 1$; collateral constraint always binds
    - Individual abilities evolve according to standard diffusion process. Then stationary abilities distribution is a Gamma distribution, to which we can compare the stationary wealth distribution.
- The unconstrained first-best allocation is a version of Buera (2009) with *only* the most able entrepreneur holding *all* the wealth.

Wealth Distribution and Capital Misallocation

The collateral constraint leads to

Definition

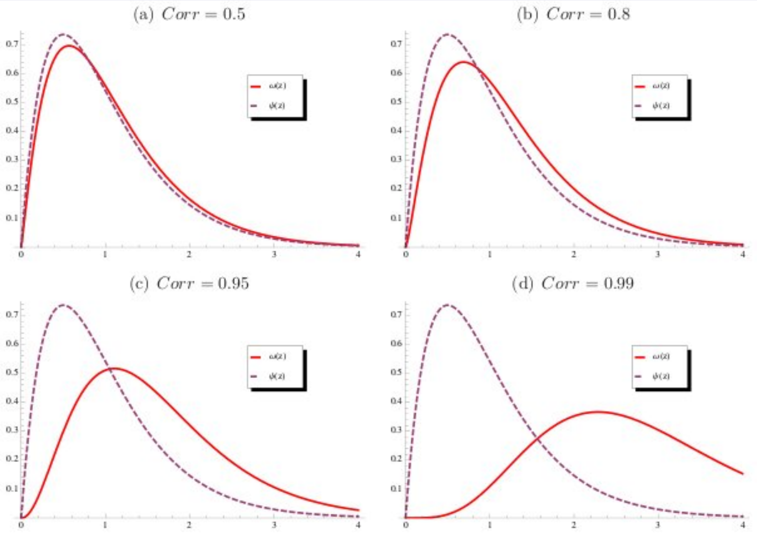
Capital misallocation: a Pareto improvement is possible by shifting capital among agents.

- Without constraint, we would achieve the first-best by FWT
- Most efficient state of the world is perfect persistence (recall earlier exercise)

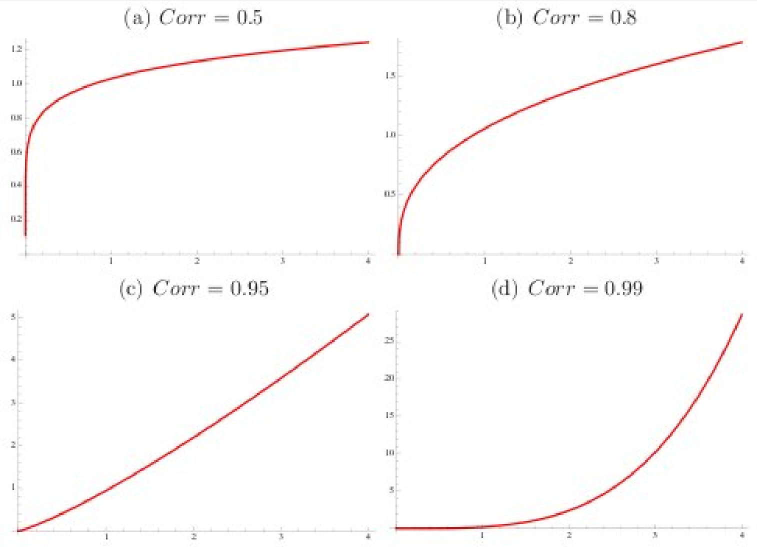
i.e. Endogenous TFP is highest when ability correlation is perfect, since high types can accumulate faster

$\Rightarrow$ More unequal wealth distribution is more efficient

c.f. This is the dynamic version of the intuition we got from the static Roy model



Ability and wealth distributions (Moll, 2014).



Expected wealth levels by ability (Moll, 2014).

Endogenizing the Collateral Constraint I

Usually modeled as imperfect enforcement of borrowing contracts. There are several ways to endogenize the constraint.

One way: vary the punishment following default, e.g.

- Cagetti and De Nardi (2006) assume entrepreneurs lose ϕ of their total assets, be worker for one period, and return to the financial market.
- Buera et al. (2011) assume they can return immediately to the financial market.

c.f. *in many limited commitment models, punishment is autarky forever.*

Endogenizing the Collateral Constraint II

More realistically: Possibility of Default

- The no autarky or eternal autarky cases are relatively easy to analyze - but finite, positive time exclusion from financial markets are only analyzable numerically in general
- Equilibrium default is pretty hard to analyze, even numerically.

Endogneous Collateral Constraint with No Exclusion

- For now, no equilibrium default with no autarky.
- Quantitative, discrete time version of previous model: abilities change with probability $1-\gamma$
- An entrepreneur loses ϕ of his wealth upon default (that is the only punishment). Then
- Value of Default:

$$V_d(k;e;\phi) = \max_{c,a'}\{u(c) + \beta\left[\gamma V(a',e) + (1-\gamma)\mathbb{E}_{e'}V(a',e')\right]\}$$

$$c + a' \leq (1-\phi)\left[\max_{l}\{ef(k,l) - wl\} + (1-\delta)k\right]$$

- Value of no Default:

$$V(k;a,e) = \max_{c,a'}\{u(c) + \beta\left[\gamma V(a',e) + (1-\gamma)\mathbb{E}_{e'}V(a',e')\right]\}$$

$$c + a' \leq \max_{l}\{ef(k,l) - wl\} - Rk + (1+r)a$$

Endogneous Collateral Constraint with No Exclusion

Clearly, $V \geq V_d$ (no equilibrium default) if

$$\max_l \{ef(k,l) - wl\} - Rk + (1+r)a$$
$$\geq (1-\phi)\left[\max_l \{ef(k,l) - wl\} + (1-\delta)k\right]$$
$$\Rightarrow \quad (1+r)a = (1-\phi+r+\phi\delta)\hat{k} - \phi \max_l \{ef(\hat{k},l) - wl\}.$$

- Define $\hat{k} \equiv \hat{k}(a,e;\phi)$, the maximum possible debt
- Only depends on current (a,e)
- $\hat{k}(\cdot)$ is increasing in all arguments

Some Numerical Considerations

The typical financial constraint is notoriously easy to solve:

1. Same as Aiyagari (1994), with additional loop over w
2. Inside R-loop, make w-loop
   - For guess of w, compute state-dependent incomes $y(a, e)$
   - Given occupation choice, check labor market clearing
3. Iterate to convergence (simple Newton or Brent will do)

If constraint is endogenous and not analytically characterized,

1. Inside w-loop, functionally approximate $\hat{k}$:
   - Given parametric guess, compute state-dependent incomes $y(a, e)$
   - Given incomes, compute whether default is preferred
   - Update till never default
2. Iterate to convergence

Some Numerical Considerations

Endogenous constraint may be costly nested loops:

- If we know $\hat{k}(a, e)$ is increasing in (a, e), why not just

  $$\hat{k} = \phi_a a + \phi_e e$$

  or something similar?
- Reduced-form representation of limited commitment: our "λ" is one such approximation
- Will never matter for calibration
- Usually won't matter for counterfactuals either...varying ϕ is almost same as varying λ
- Unless default itself is of interest, not really worth it

Endogneous Collateral Constraint, General

- In general, two things to consider:
- What is the period-to-period punishment?
(usually a punishment $p(X)$ on period income or assets)
- How long are you punished? (n periods)
- An agent facing default at time t chooses

$$V(X,t) = \{V_S(X,t), V_D(X,t)\}$$

- X are whatever state dictated by the model and assumptions

Endogneous Collateral Constraint, General

- The default value is modeled as

$$V_D(X,t) = \max \left\{ \sum_{j=0}^{n-1} \beta^j \mathbb{E} u(c_{j+t}) + \beta^n \mathbb{E} V(X(t+n), t+n) \right\}$$

- Period punishments are modeled as

$$c + a' = (1 - p_y(X))y(X) + (1+r)(1 - p_a(X))a$$

- X can include anything; not just usual state but also length of time spent in default state, when you entered default state, etc.
- Most interesting applications (e.g. sovereign debt, consumer bankruptcy) *will* include such objects, which explodes the state space and makes problem difficult

Endogneous Collateral Constraint, General

- If $n > 0$, period punishments can be modeled differently for every period in default
- When $n > 0$, the interest rate in the default state $r(X)$ can change too.
- Depending on model, we can endogenize the default punishments $(p_y(X), p_a(X), r(X))$, so that V_D is never chosen
- The opposite extreme standard case of what we did today is no period punishment, but forever in default state

$$
\begin{aligned}
V_D(y, a) = \max \left\{ u(c) + \beta \mathbb{E} V_D(y', a') \right\} \\
c + a' = y + (1 + r)a \\
a' \geq 0
\end{aligned}
$$

e.g., Zhang (1997)

Misallocation, Distortions and TFP

- Instead of explicitly modeling distortions, we can quantitatively capture misallocation effects with reduced form parameters in the firms' f.o.c.'s:

$$\pi_e = \max_{k,l} (1 - \tau_Y(e)) f(e, k, l) - wl - (1 + \tau_K(e)) Rk,$$

- For each potential entrepreneur with ability e, $\{\tau_X\}_{X=Y,K}$ are "ability level"-specific:
  - $\tau_K(e)$: capital market distortions
  - $\tau_Y(e)$: everything else
- In such accounting exercises, only two of three distortions identified (can include $\tau_L(e)$, but then must take out one of the other two)

Intensive and Extensive Misallocation

The thought experiment is:

1. Keeping K constant, can we increase aggregate output? (static, intensive)

2.i If we could measure $e, \{\tau_X(e)\}$, how are firms in operation affected (dynamic, intensive), and

2.ii which firms are (not) in operation because of distortions (dynamic, extensive)?

Note that this is a *purely quantitative* exercise.

1. ..is Restuccia and Rogerson (2008); Hsieh and Klenow (2009); Bloom et al. (2012)

2.i ...is Moll (2014); Buera et al. (2011); Buera and Shin (2013)

2.ii Explicit extensive margin: Buera et al. (2011); Midrigan and Xu (2013)

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, August 1994, *109* (3), 659–684.

Angeletos, George-Marios, "Uninsured idiosyncratic investment risk and aggregate saving," *Review of Economic Dynamics*, 2007, *10* (1), 1 – 30.

Benhabib, Jess, Alberto Bisin, and Shenghao Zhu, "The Wealth Distribution in Bewley Models with Investment Risk," NBER Working Papers 20157, National Bureau of Economic Research, Inc May 2014.

Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts, "Does Management Matter? Evidence from India," *The Quarterly Journal of Economics*, 2012.

Buera, Francisco, "A dynamic model of entrepreneurship with borrowing constraints: theory and evidence," *Annals of Finance*, June 2009, *5* (3), 443–464.

Buera, Francisco J. and Yongseok Shin, "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 2013, *121* (2), 221 – 272.

_ **, Joseph P. Kaboski, and Yongseok Shin**, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, August 2011, *101* (5), 1964–2002.

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 2006, *114*, 835–870.

Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull, "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 2003, *111*, 818–857.

Gabaix, Xavier and Augustin Landier, "Why Has CEO Pay Increased So Much?," *Quarterly Journal of Economics*, 2008, pp. 49–100.

Goos, Maarten, Alan Manning, and Anna Salomons, "Explaining Job Polarization: Routine-Biased Technological

Change and Offshoring," *American Economic Review*, 2014, *104* (8), 2509–26.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, November 2009, *124* (4), 1403–1448.

Huggett, Mark, "The Risk Free Rate in Heterogeneous-Agent, Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 1993, *17*, 953–969.

Lee, Eunhee, "Trade, Inequality, and the Endogenous Sorting of Heterogeneous Workers," 2015.

Lucas, Robert E., "On the Size Distribution of Business Firms," *Bell Journal of Economics*, Autumn 1978, *9* (2), 508–523.

Midrigan, Virgiliu and Daniel Yi Xu, "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 2013.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?," *American Economic Review*, 2014.

Quadrini, Vincenzo, "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics*, 2000, *3*, 1–40.

Restuccia, Diego and Richard Rogerson, "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," *Review of Economic Dynamics*, October 2008, *11* (4), 707–720.

Tervio, Marko, "The Difference That CEOs Make: An Assignment Model Approach," *American Economic Review*, June 2008, *98* (3), 642–68.

Zhang, Harold H, "Endogenous Borrowing Constraints with Incomplete Markets," *Journal of Finance*, December 1997, *52* (5), 2187–2209.