Life-cycle Inequality and Ben-Porath

DEEQA Quantitative Macro

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Ben-Porath Model

1. Human capital accumulation model
2. Contrasts with the reduced form "Mincer Model": *quality* of human capital
3. Typically used to life-cycle earnings profiles
   * References: Ben-Porath (1967); Heckman et al. (1998); Guvenen (2007); Huggett et al. (2011); Manuelli and Seshadri (2014)

- *Most of the theoretical literature comes from classical labor economics. For macroeconomic roots, see Uzawa (1965); Lucas (1988)*

No Time For:

- Learning: learning by doing, learning about yourself
  - Assumes that human capital accumulation occurs by working (in contrast to not working)
  - Learning about *yourself* is different from asymmetric information
  - Stange (2012); Sanders (n.d.)
- Obvious appeal of LBD: quantitatively simpler to implement (close to reduced form)
- As usual, data suggests a mixture of LBD and OJT (Heckman et al., 2002)

Also no time for:

- Heterogeneous human capital: general vs occupation/firm specific human capital
  - College majors, different jobs, different skills, etc.
  - See works from Joe Altonji, Joe Hotz, Michael Keane, Chris Taber, Kenn Wolpin, etc.
- Asymmetric information: adverse selection in college admissions, job market
  - Job search/matching with heterogeneous skill levels is basically a Roy model with reduced-form frictions
- Human capital risk: human capital investment under uncertainty
  - See works from Eaton and Rosen (1980), Tom Krebs (his career!) and yours truly...really obvious yet underdeveloped area

Motivating Data for the Ben-Porath Model

1. Ben-Porath model can explain schooling, OJT and earnings profile (Heckman et al. (1998); Huggett et al. (2011); Manuelli and Seshadri (2014)
2. Not surprisingly, earnings increase rapidly early in the lifecycle, and taper off toward the end
3. Perhaps surprisingly, earnings tend to “fan out” across individuals along the lifecycle

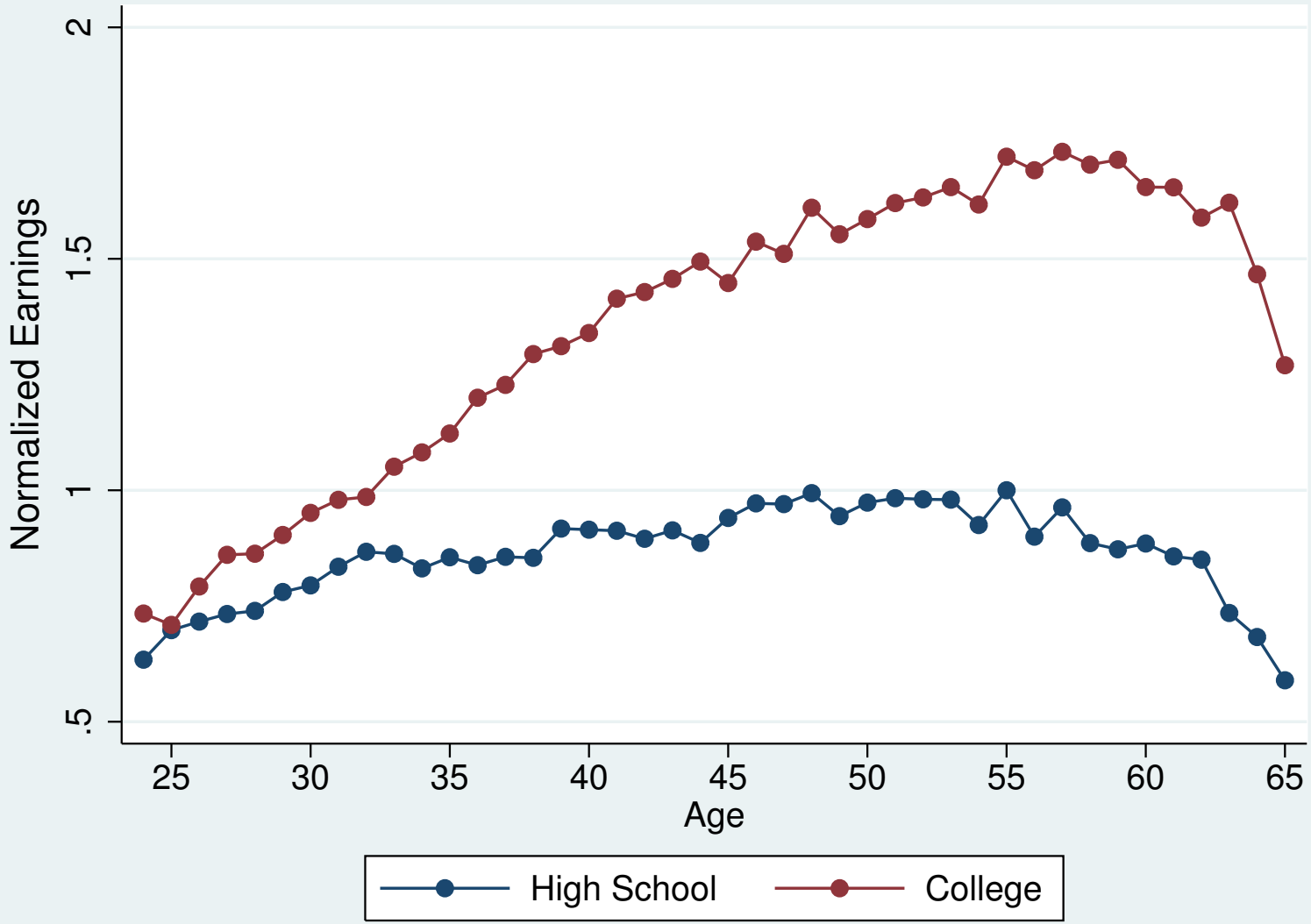
Fixed Effects Panel Regression

- Consider the APC regression

$$\log e_{i,t} = A_a + C_c + T_t + \epsilon_{i,t}$$

- Canonical identification problem: all three are not separately identified since $A_a + C_c = T_t$
- For now, let's take the time effects view
- Life-cycle earnings profile in the PSID, separately for high school and college
- PSID: best data source for individual/family data
- German equivalent, G-SOEP, also frequently used; unfortunately, I'm not familiar with French data sources (yet)

Age Earnings Profiles (PSID, heads of households)



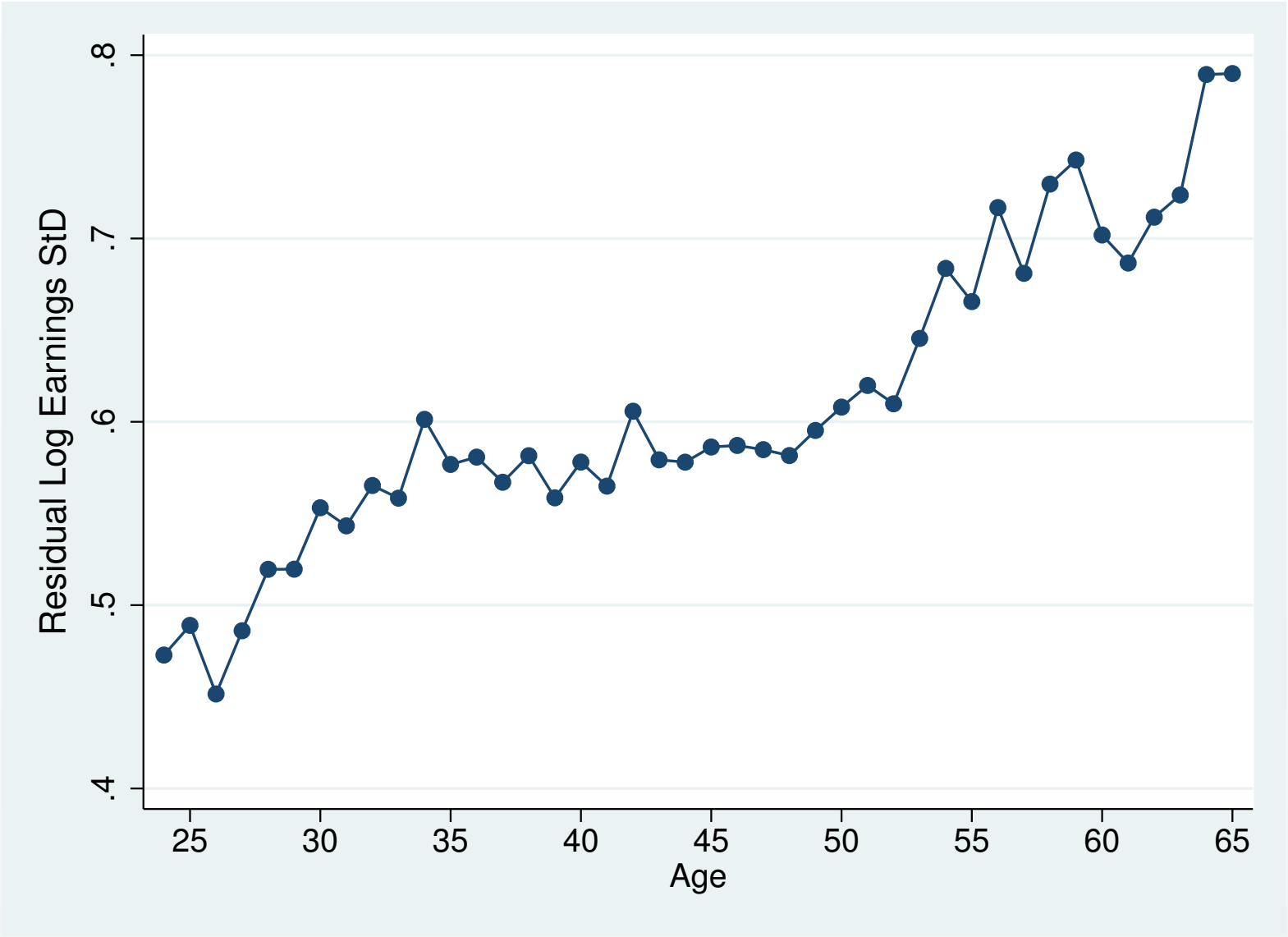
Life-cycle Inequality

- Shape of profile is perhaps not surprising
- More surprising is the "fanning out" between high school and college
- Even more surprising is the "fanning out" *within* high school and college: consider the regression

$$\log e_{i,t} = E_e + A_{e,a} + T_{e,t} + \epsilon_{i,t}$$

  that is, age-time effects regressions interacted with high school / college effects
- Plot residual earnings inequality by age

Life-Cycle Inequality (PSID, heads of households)



Some History (basically, Heckman's Career)

- Level of inequality differs, but fanning out is confirmed in most data sets, countries, etc.
- **Big Question: selection or treatment?**
- Most studies conclude it's mostly selection (exact estimates vary from 65-90%)
- Hence, a lot of research focuses on college (*before* labor market entry)
- Turns out, in the U.S., college seems to be mostly selection too...at least before recent surge in college costs (some evidence that this is not the case in France/Germany)
- Even if predetermined, still important to understand what happens over the life-cycle

Why Ben-Porath?

- Canonical Mincer (Regression) Model, still ubiquitously used:

$$\log W = \alpha + X\beta + \gamma_0 \text{EXP} + \gamma_1 \text{EXP}^2 + \epsilon$$

X : individual characteristics, observable in the data

- Can get simple age-experience effect from the data...
- Can also mechanically get "fanning out" by interacting observables, but
  - No structure between observables
  - Difficult to account for unobservables ("quality")
- To explain fanning out, need either a natural experiment or parametric structure

Canonical Ben-Porath Model

$$V(6,h_6) \equiv \max_{n_t,m_t} \left\{ \int_6^R e^{-r(a-6)} \left[wh_a(1-n_a) - m_a\right] da \right.$$

$$\left. = \int_6^R e^{-r(a-6)} [\underbrace{wh_a}_{\text{returns to human capital}} - \underbrace{(wh_a n_a + m_a)}_{\text{cost of hc production}}] da \right\}$$

$$\text{s.t.} \qquad \dot{h}_a = \underbrace{a(n_a h_a)^{\gamma_1} m_a^{\gamma_2}}_{\text{human capital production}}$$

$$n_a \in [0,1], \quad h_6 \quad \text{given}$$

- a: individual specific ability to **learn**
- h_a: human capital today, ability to **earn**
- n_a, m_a: time and good inputs into human capital; fixed time endowment every period
- w: wage rate (constant)
- zero depreciation of human capital

Characteristics of the Model

- Decreasing returns to human capital investment ($\gamma = \gamma_1 + \gamma_2 < 1$)
- Both time and goods inputs
- Tradeoff between work(n), saving(m) and hc accumulation
- Fixed age retirement
- No uncertainty environment - so equivalent to a utility maximization problem with natural borrowing constraint

HJB Equation

- HJB equation for the lifetime income maximization problem:

$$rV(a,h) - \frac{\partial V(a,h)}{\partial a}$$
$$= \max_{n,m} \left\{ wh - (whn + m) + \frac{\partial V(a,h)}{\partial h} \cdot a(nh)^{\gamma_1} m^{\gamma_2} \right\}$$

- Refer to my paper with Roys and Seshadri (2016) online appendix for the complete solution (on web)
- Today consider a discrete version

Discrete Time Model

$$\mathcal{L}(\Lambda) = \sum_{a=6}^{R} \left(\frac{1}{1+r}\right)^{a-6} \Big\{ wh_a - (wh_a n_a + m_a) + \lambda_a \Big[a(n_a h_a)^{\gamma_1} m_a^{\gamma_2} - (h_{a+1} - h_a) \Big] \Big\}$$

- Note n_a never zero except in last period, due to Cobb-Douglas assumption (infinite marginal returns)
- First consider interior solution when $n_a < 1$

Interior Solution

- Define cost of hc production

$$x_a \equiv wh_a n_a + m_a$$

- Interior solution for (n_a, m_a) in terms of x_a is

$$\frac{wh_a n_a}{\gamma_1} = \frac{m_a}{\gamma_2} = \frac{x_a}{\gamma}$$

- So distinction of (n_a, m_a) doesn't really matter when $n_a < 1$

Solution to the model

Conditions for optimality are (for interior solution)

$$1 = \lambda_a \cdot a\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\left(\frac{\gamma}{x_a}\right)^{1-\gamma}$$

$$\lambda_a = \frac{w + \lambda_{a+1}}{1 + r}$$

- λ_a: marginal value of human capital

1. MC = Marginal value$\times$ Marginal increase in human capital
2. Marginal value equalized between today and tomorrow

$\Rightarrow$ increase in wage plus marginal value of human capital tomorrow, discounted to today

Solution to the model

- λ_a is (weakly) declining over time if:
  1. w is large enough (direct returns to human capital)
  2. r is low enough (borrowing against future earnings is cheap)

  All makes sense.
- Otherwise, hc accumulation never occurs - we won't look at this case

"Schooling"

- Now call the length of the life-cycle that $n_a = 1$ "schooling time," s
- In schooling period, law of motion is given by

$$1 = \lambda_a \cdot \frac{\gamma_2 a h_a^{\gamma_1}}{m_a^{1-\gamma_2}}$$

$$\lambda_a = \frac{\lambda_{a+1}}{1+r} \cdot \left(\frac{\gamma_1 a m_{a+1}^{\gamma_2}}{h_{a+1}^{1-\gamma_1}} + 1 \right)$$

Solution to the model (cont.)

- We need two boundary conditions to "match and paste" the two pairs of equations
- These are analogous to value-matching and smooth-pasting conditions in continuous time:

$$\lambda_s = \frac{1}{1+r}\left(w + \lambda_{s+1}\right)$$

$$\frac{x_s \gamma_1}{\gamma w h_s} \geq 1 \quad \text{and} \quad \frac{x_{s+1}\gamma_1}{\gamma w h_{s+1}} < 1$$

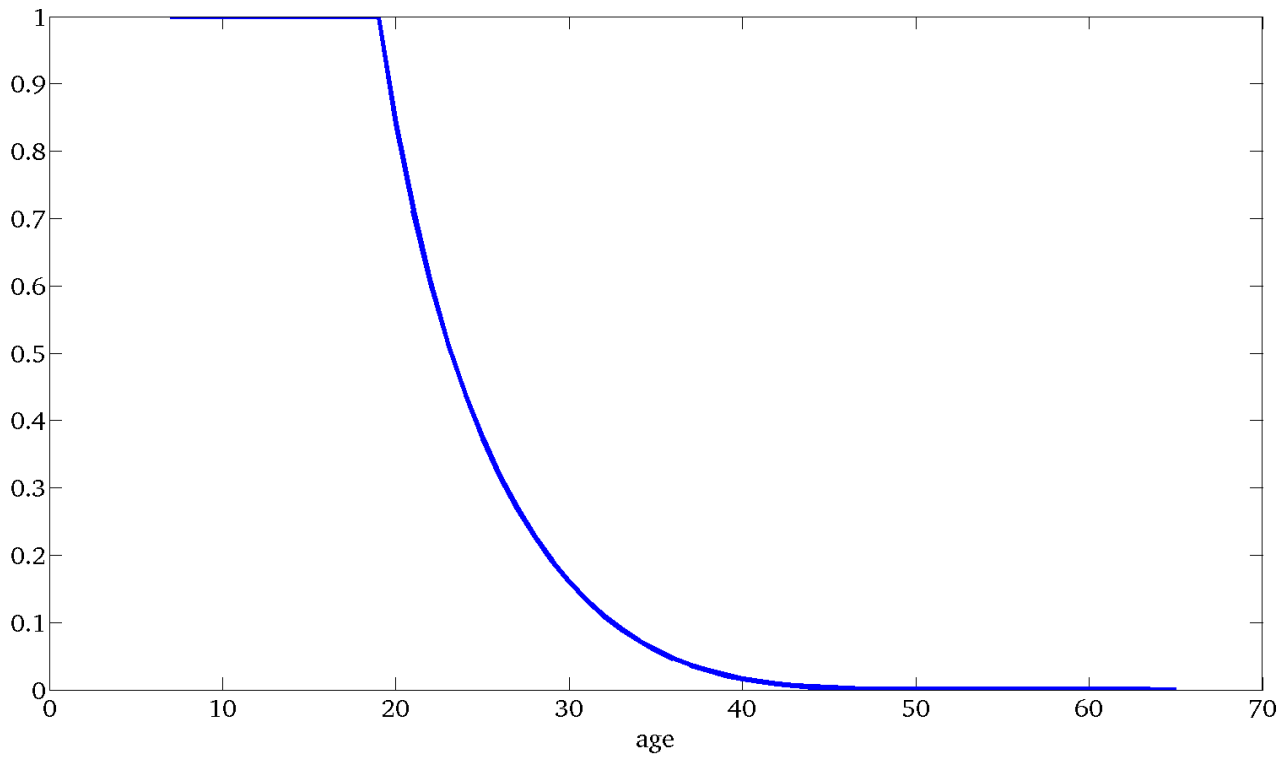
(x_s is the total cost you would choose *without* the $n \leq 1$ constraint.) These pin down s and $h(s)$

- Closed form solution in continuous time is more elegant: VM simply means $n = 1$, and SP is $\frac{\partial V(a,h)}{\partial h}$ at $n = 1$

Implications of the Model

- x_a, and hence $h_a n_a$, m_a decline over time
  - As long as you retire (die), value of accumulating declines (less time to reap the returns)
- h_a increases over time, at least initially in the life-cycle
- Implies that n_a declines fast
- However, note that there is no decumulation/depreciation

Empirical Implications - Schooling



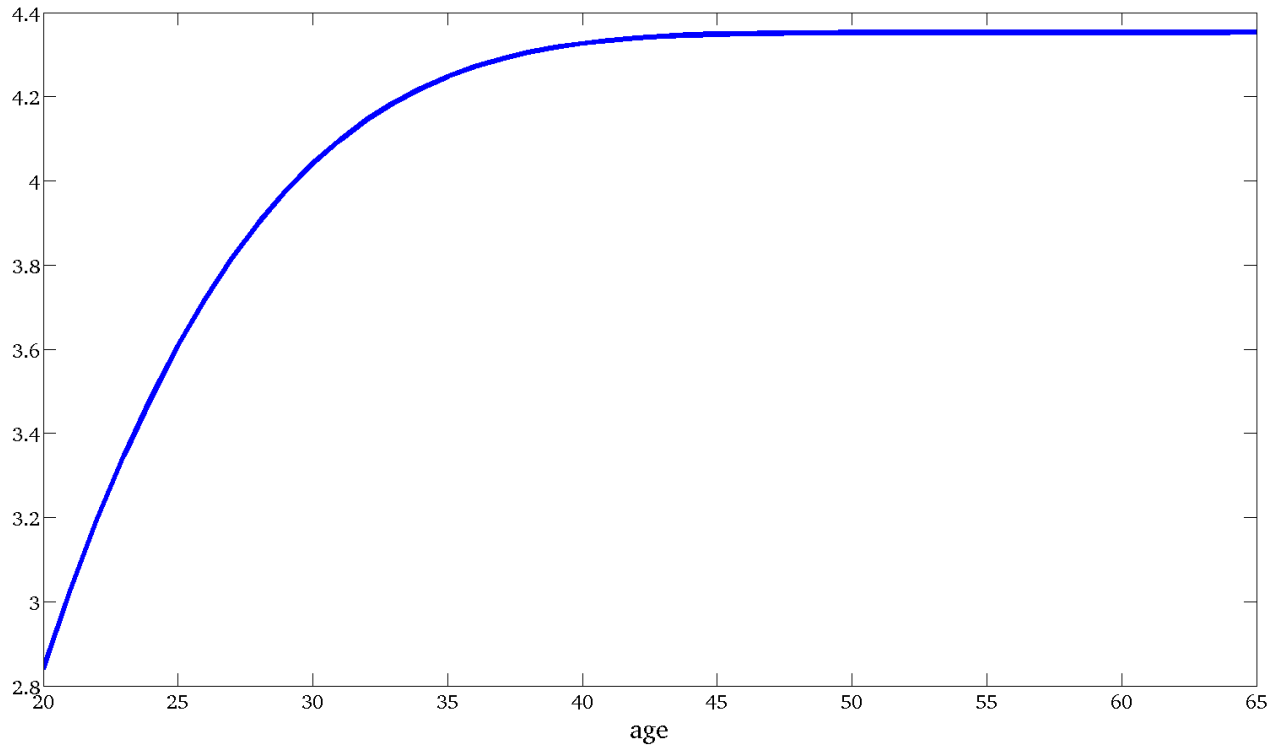
- Parameters chosen to matched to 13 years of schooling:

$$h_6 = 1, a = 0.1036, \gamma_1 = 0.63, \gamma_2 = 0.3$$

- Wages and interest rate implied from neoclassical firm:

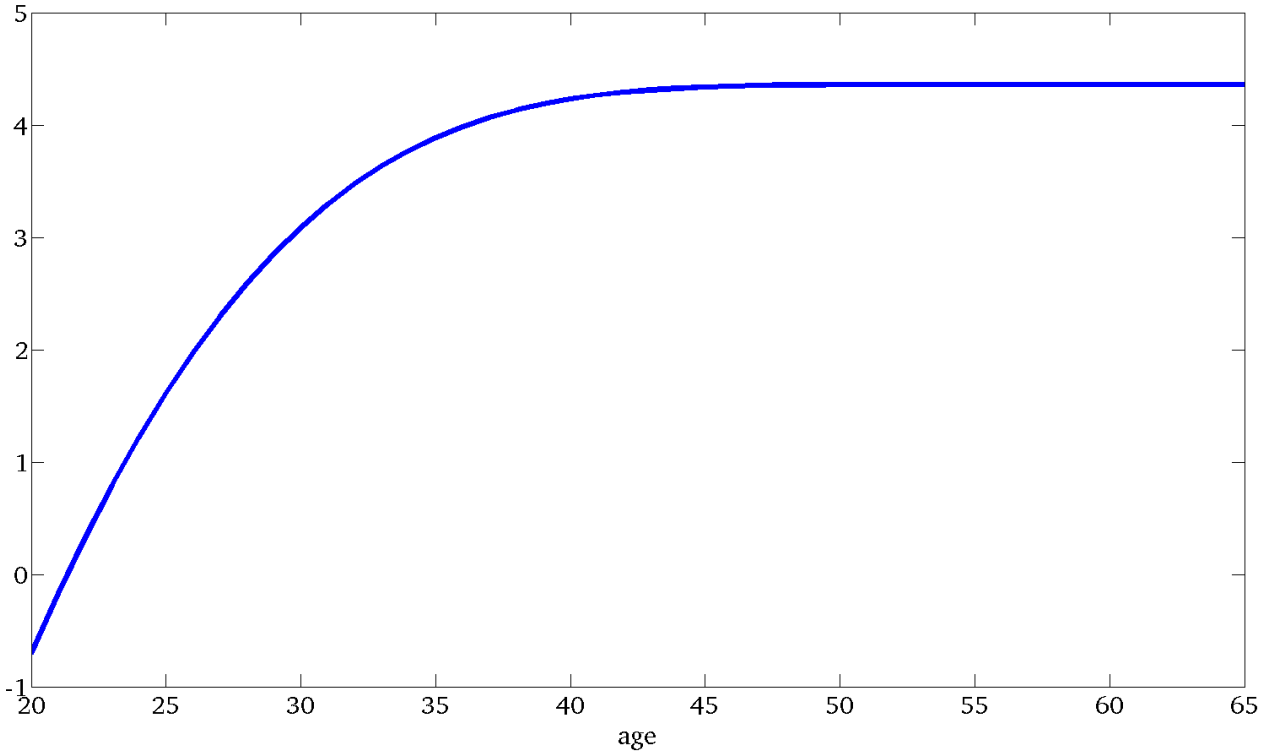
$$r = 0.04, \alpha = 0.3, \delta = 0.09$$

Empirical Implications - Wage Profile



- Only empirical evidence of human capital are wages
- We assumed $\delta_h = 0$: wages don't fall at end of life-cycle
- a: slope effects, h_6: level effects (Huggett et al. (2011))

Empirical Implications - Earnings



- $wh(1-n) - \pi m$: "measured earnings"
- Employer deducts training expenses from wages
- $\pi = 1$ is conventional, but not always

So What?

- Initial heterogeneity in (a, h) can explain 65-90% of inequality (Keane and Wolpin, 1997; Huggett et al., 2011)
- That is, shocks that happen later in life don't matter much
- Note that learning abilities, a, determines the slopes of the profiles, so variance in a implies fanning out
- Turns out though, that the level differences in h (along with its correlation with a) are the main determinant of inequality Heckman et al. (1998); Huggett et al. (2011)
- Pure statistical models also conclude that much of wage variance seems permanent, volatility plays small role (e.g. Chen (2008))

Other Extensions

- Time and goods input distinction doesn't matter much mathematically, but can use to match years of schooling, OJT, expenses, etc. (Manuelli and Seshadri (2014))
- Can endogenize retirement age (in various ways), typically requires some form of taste for leisure; also related to labor-leisure choice (Hazan (2009); Manuelli et al. (2011))
- Most extensions include some form of non-hc related of uncertainty (usually in the returns) (Huggett et al. (2011))
- Shocking the human capital accumulation production function itself, not the returns, has large effects (Krebs (2007))
  - even larger if possible to decumulate human capital (Krebs (2003))

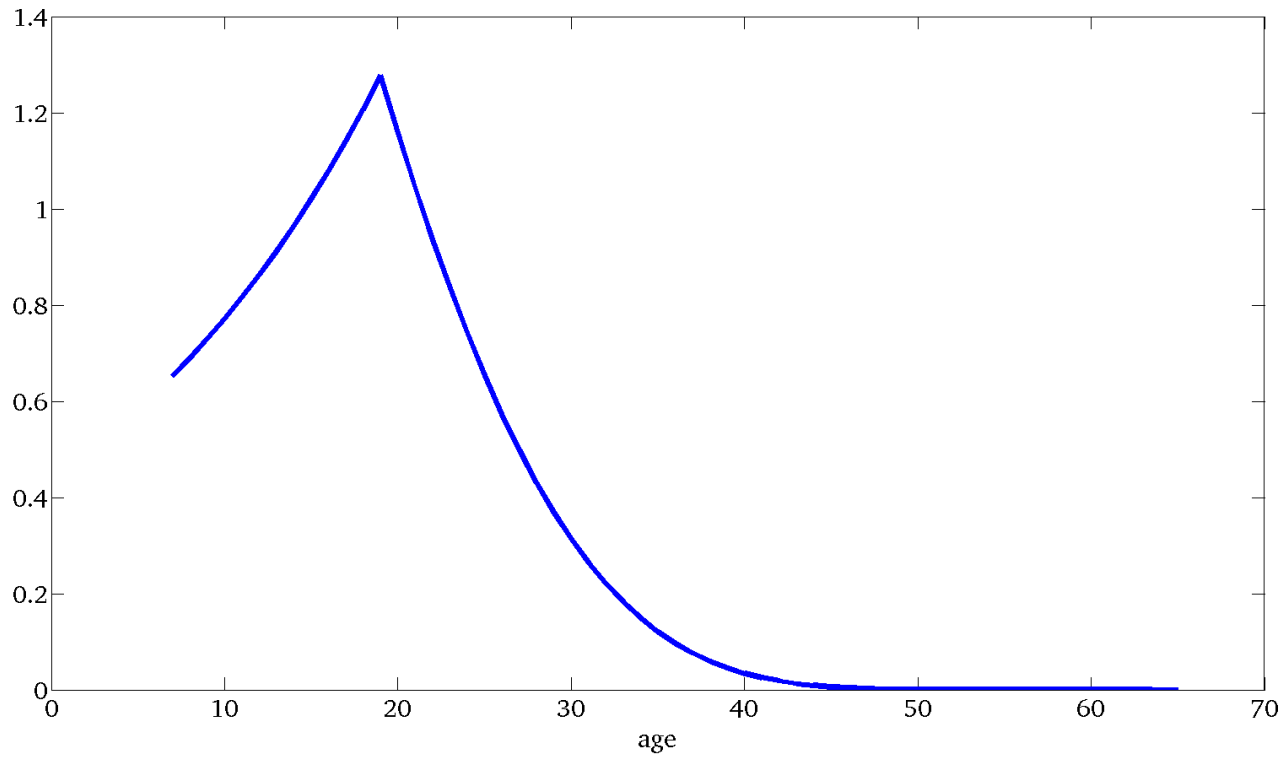
Dynamic Complementarity in Ben-Porath

- Note that there is no DC after schooling: this is because the cost of accumulation involves forgone wages
- But it matters during schooling:

$$\frac{\delta^2 f}{\delta h \delta x} = \frac{\gamma_1 \gamma_2}{h^{1-\gamma_1} m^{1-\gamma_2}} > 0$$

- Optimal response is to invest more later in education!

Dynamic Complementarity in Ben-Porath



- When $n = 1$, m is increasing in h
- Also raises the question of whether early or later subsidies are more important!

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