Endogenous Technology Growth

Sang Yoon (Tim) Lee

Toulouse School of Economics

Jan 30-Feb 6, 2018

last updated: December 31, 2017

Review from Last Week

- Wrapped up human capital extension of Solow model
- Technology extension of Solow model
- Discussed characteristics of ideas: increasing returns, non-rivalry, excludability

Plan for today and ahead

1. Extend technology growth to Romer model
   ("endogneous growth," today)
2. International transfers: Capital or Technology?
3. Combine human capital and technology into one model

Technology Growth as "Idea Production"

Before we go further...what is an idea?

- Rivalrous
- Excludable
- Fixed Costs and Increasing Returns: e.g., $f(x) = 100(x - F)$

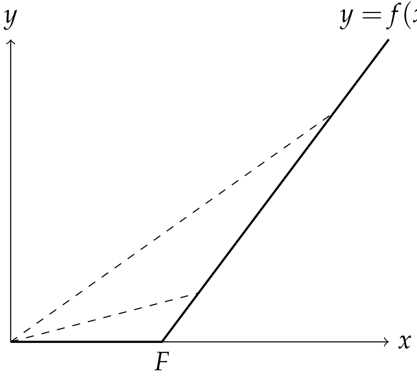
Rivalrous vs. Excludable Goods

	Rivalrous Goods	Nonrivalrous Goods
High	Lawyer services	Satellite TV
	Cellphone, USB drive	Cell phone service
⇑		
Degree of		Computer Software
Excludability		Retail management
⇓		
	Fish in the sea	Research
Low	Fruit in the forest	National Defense

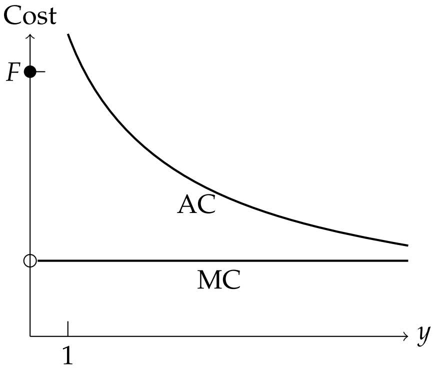
Fixed Cost and Increasing Returns

Example:

$$f(x) = \max(100(x - F), 0)$$



Productivity



Average and Marginal Costs

The Idea Production Function: Pros and Cons

- In class we will assume that ideas are produced according to

$$\dot{A} = \eta L_A^{\lambda} A^{\phi}$$

ϕ : scale of idea spillovers $\Rightarrow$ larger stock of ideas create more ideas

λ : decreasing returns in idea productions (1000th scientest will not add as much as the 10th one)

- Looks innocuous, however it's not:
  - does growth from research cease with population growth...?
  - to avoid this, we could assume $\phi = 1 \Rightarrow$ **Romer Model**
  - however, this doesn't fit well empirically (but we will still learn it!)

Solow Model with Research Sector

Forget about human capital for now, and instead assume there are two sectors: "goods" production and "ideas" production. Both sectors use labor:

- L_Y: goods production
- L_A: idea production

where $L = L_Y + L_A$, and the technologies for each sector are

- Production:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}, \quad \dot{K} = sY - \delta K$$

- Ideas:

$$\dot{A} = \eta L_A^{\lambda} A^{\phi}$$

BGP Solution

- How do we solve this model? Think about what a “BGP” is:
  1. BGP is when growth rates are constant
  2. In this model, the only undetermined growth rate is technology growth
  3. Once we know g, almost same as Solow model
- Let $L_A/L = \mu$, a new parameter. Similar to the Solow model,

$$y^*(t) = \left(\frac{s}{n+g^*+\delta}\right)^{\frac{\alpha}{1-\alpha}} \times \underbrace{(1-\mu)}_{\text{only difference from Solow Model}} \times A(t)$$

- g^* no longer exogenous

Solving for g^*

- Assume g^* is constant and proceed backward to obtain an expression for it
- Exactly how we got κ (growth rate of normalized capital)
- We know that

$$g^* \equiv \frac{\dot{A}}{A} = \frac{\eta L_A^\lambda}{A^{1-\phi}}$$

$$0 \equiv \frac{\lambda \dot{L}_A}{L_A} - (1-\phi) \cdot \frac{\dot{A}}{A} = \frac{\lambda \dot{L}_A}{L_A} - (1-\phi) g^*$$

- We also know that on a BGP, L_A / L_Y must be constant! So it must be that

$$\frac{\dot{L}_A}{L_A} = n$$

- So all this leads to

$$g^* = \frac{\lambda n}{1-\phi}$$

BGP Solution

The BGP growth rate and output per worker:

$$g^* = \frac{\lambda n}{1-\phi}, \qquad \dot{A} = \eta L_A^{\lambda} A^{\phi},$$

$$y^*(t) = \left(\frac{s}{n + g^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\mu) \cdot A(t)$$

- More population, more ideas (λ)
- Ideas (partially) non-rivalrous, more ideas creates more ideas (ϕ)
- Ideas are non-excludable (everyone enjoys g^*)

$\Rightarrow$ So population increase no longer “bad"

- Instead of dumping everything on g, now dump things that we cannot explain into (λ, ϕ) to match observed level of g in the data

$\Rightarrow$ A bit more specific about the nature of “what we don’t know”

Who is making the ideas?

- Easy to use ideas to create new ideas
  (non-rivalry in idea production, positive externality)
- Everyone enjoys per-capita income growth g^*
  (non-rivalry in growth rates, positive externality)

$\Rightarrow$ **In reality, no one would want to pay for it**

$\Rightarrow$ Need some excludability

- **Romer Model** will deal with these issues

Romer Model

- Romer goes further, endogenizes μ
  - $(\lambda, \phi) = 1$: No longer have anything to "dump" on

  $\Rightarrow$ Must match both g^* **and** μ^* in the data
- Final good sector same as before
- Research sector same as before: idea production non-rivalrous
- Additional **intermediate good** sector
  - Transforms ideas into separate goods: excludable
  - **Excludability creates profits**

$\Rightarrow$ this is the **first time** we will solve for an equilibrium model

Romer Model Setup: Laws of Motion

- Simplifying assumption: no population growth

⇒ Fixed population L split between research and output:

$$L_A = \mu L, \qquad L_Y = (1-\mu)L$$

- Research sector is competitive and uses only labor:

$$\dot{A} = \eta L_A A$$

- Representative consumer as in RCK with budget constraint

$$\dot{a} = w + ra - c \quad \Rightarrow \quad \dot{\hat{a}} = \hat{w} + (r-g)\hat{a} - \hat{c}$$

* Note that (μ, g) will be endogenously determined

Romer Model Setup: Intermediate Good Sector

- Intermediate good sector buys one unit of an idea to produce differentiated "capital good" (or "idea good")
- Ideas transform (raw) capital into differentiated "capital goods":

$$x_j = x(k_j) = k_j$$

- Raw capital is same for everyone, but not the transformed goods
- Makes ideas excludable through rivalrous capital production
- Each little firm is **monopolistic**
    - positive profits make using ideas worthwhile

Romer Model Setup: Final Good Sector

- Final goods sector is competitive and uses labor and ***all*** differentiated capital goods as inputs:

$$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha dj$$

A: Number of capital goods in the economy - size of "ideas" or "inventions"

x_j: the amount of input from each type of capital good, produced by a monopolist

Solving the Romer Model

Assuming we are on a BGP, we solve for the **equilibrium**.
Remember:

- Research and final goods sectors are competitive, but intermediate goods sector is monopolistic
- Final good price is normalized to 1
- Price of final good must include
  - price of the raw capital needed for intermediate good production... **but also**
  - price of the idea embedded in each capital good
- An **equilibrium** is defined in terms of prices and quantities - up to now, we have ignored prices

Final goods producer's problem

The final goods producer solves

$$\max_{L_Y, x_j} \left\{ L_Y^{1-\alpha} \int_0^A x_j^\alpha dj - wL_Y - \int_0^A p_j x_j dj \right\}$$

where

- p_j: price of type j capital - final goods sector takes as given
- w: wage rate of labor

and the solution is

$$w = (1-\alpha) \cdot Y / L_Y$$
$$p_j = \alpha \cdot \left(L_Y / x_j \right)^{1-\alpha}.$$

So whatever price p_j the monopolist sets, the price elasticity with respect to demand is

$$\frac{d \log p_j}{d \log x_j} = \frac{p'(x)x}{p(x)} = -1 + \alpha$$

Intermediate Goods Sector

- Each capital good producer has monopoly rights, so s/he can set his/her own price - solves

$$\max_{x_j} \left\{ p_j(x_j) x_j - R x_j \right\}$$

where s/he knows that

$$p_j = \alpha \left(L_Y / x_j \right)^{1-\alpha}$$

Solution is

$$\begin{aligned} p'(x)x + p(x) &= R \\ \Rightarrow \quad p &= R/\alpha \end{aligned}$$

since $\alpha < 1$, this is a "markup"

- Since all intermediate firms charge the same price, $x_j = x$ for all j

Definition of BGP equilibrium

A BGP equilibrium is a set of prices $\{w, r, R, p_j, P_A\}$ and allocation $\{c, K, L_Y, L_A, x_j\}$ such that:

1. Given (w, r), representative consumer maximizes utility
2. Given (w, p_j), final goods producers maximize profit
3. Given $(p_j(x_j), R)$, intermediate goods producers maximize profit
4. Given (w, P_A), research firms maximize profit
5. Markets clear (can ignore goods markets):

$$L_Y + L_A = L$$
$$\int_0^A k_j dj = \int_0^A x_j dj = K$$
$$\eta L_A A = \dot{A}$$

6. All quantities grow at a constant rate

Simplifications

1. Since $x_j = x$ for all j, aggregate amount of capital

$$K = \int_0^A x_j dj = Ax$$

2. Substituting this in the goods production function, we get

$$Y = L_Y^{1-\alpha} A x^\alpha = K^\alpha (AL_Y)^{1-\alpha}$$

Back to the original technology!

Implications: Factor Payments

1. From the final good producer's demand for x,

$$pAx = \alpha(Ax)^{\alpha}(AL_Y)^{1-\alpha}$$
$$\Rightarrow \qquad RK = \alpha^2 K^{\alpha}(AL_Y)^{1-\alpha} = \alpha^2 Y$$

2. Note that

$$RK + wL_Y = \alpha^2 Y + (1-\alpha)Y < Y$$

3. Since APF is HD1, payments to each factor should sum up to one
4. Means some factor other than capital and labor are getting paid

Implications: Returns to Ideas

- Each monopolist earns

$$\pi = (p - R)x = \frac{1-\alpha}{\alpha} \cdot \frac{RK}{A} = \frac{\alpha(1-\alpha)Y}{A}$$

  and there are exactly a mass A of them

- The profits are the returns to an idea

Interim Review of Where We Are:

1. A final goods producer profit maximization gives us

$$w = (1-\alpha)\cdot Y/L_Y, \qquad p_j = \alpha \cdot \left(L_Y/x_j\right)^{1-\alpha}$$

2. The intermediate goods producers' profit maximization gave us

$$p = R/\alpha$$

3. and because $x_j = x$ for all j, by capital market clearing:

$$K = \int_0^A x_j dj = Ax, \qquad Y = L_Y^{1-\alpha} A x^\alpha = K^\alpha (AL_Y)^{1-\alpha}$$

4. and monopoly profits for an intermediate good producer was

$$\pi = (p-R)x = \frac{1-\alpha}{\alpha}\cdot\frac{RK}{A} = \frac{\alpha(1-\alpha)Y}{A}$$

Price of Ideas

- For research sector to exist, there must be *prices*
- Can infer the price of ideas from the intermediate firms' profits
- This is obtained by arbitrage:
  1. Suppose I spend P_A to buy an idea. Then I can earn monopoly profits, and sell it next period
  2. Otherwise, I can save it and earn rP_A, so

$$rP_A = \pi + \dot{P}_A \quad \Rightarrow \quad r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A}$$

BGP Price of Ideas

- On a BGP
  1. r must be constant
     - therefore, π and P_A must grow at same rate
  2. Note that $\pi = (p - r)x$ and $p = \alpha \left(L_Y/x\right)^{1-\alpha} = R/\alpha$
     - so $p - r$ is constant
     - hence π grows at same rate as x, which grows at same rate as L_Y
  3. So (P_A, π, x, L_Y) all have the same growth rate of $n = 0$!

- Consequently from $r = \pi/P_A$,

$$P_A = \pi/r = \frac{\alpha(1-\alpha)}{r} \cdot \frac{Y}{A}$$

- Makes sense: price of idea is discounted sum of profits

$$P_A(t) = \int_t^\infty e^{-r(\tau - t)} \pi(\tau) d\tau = \pi/r$$

since we assume a BGP, and π is constant

Research Sector

- By arbitrage, wage rates must be same in all sectors. First solve

$$\max_{L_A} \left\{ P_A \cdot \dot{A} - wL_A = (\eta P_A A - w)\, L_A \right\}$$

so $w = \eta P_A A$

- In equilibrium this must equal the wage rate in the production sector (*labor market clearing*), so

$$\begin{aligned}
\eta P_A A &= \frac{(1-\alpha)Y}{L_Y} \\
\frac{\eta\alpha(1-\alpha)}{r} \cdot \frac{Y}{A} \cdot A &= \frac{(1-\alpha)Y}{L_Y} \\
\Rightarrow \qquad (1-\mu)L &= r/\eta\alpha \\
\mu &= 1 - r/\eta\alpha L
\end{aligned}$$

BGP Labor Share and Growth Rate

- We know μ, the research labor share. If we find r, we are done!
- From households problem we know that on a BGP,

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\gamma} \cdot [r - g - \hat{\rho}] = 0 \quad \Rightarrow \quad r = g + \hat{\rho} = \gamma g + \rho$$

- Using this, we obtain the following system in (μ, g):

$$\begin{cases} \mu = 1 - (\gamma g + \rho)/\eta \alpha L \\ g = \eta \mu L \end{cases} \quad \Rightarrow \quad \begin{cases} \mu^* = \frac{\alpha - \rho/\eta L}{\alpha + \gamma} \\ g = \frac{\alpha \eta L - \rho}{\alpha + \gamma} \end{cases}$$

Implications

- Solution: more reseachers, faster technological progress

$$\mu^* = \frac{\alpha - \rho/\eta L}{\alpha + \gamma}, \quad g^* = \frac{\alpha\eta L - \rho}{\alpha + \gamma}$$

- More research if
  1. More efficient in research (large η): obvious
  2. More patient (small ρ), less risk-averse (higher EIS, low γ):
     care about progress for tomorrow
  3. More people (large L): larger market, more profits
  4. Larger α: more profits but also more competition (less markup)

c.f. $1/\gamma = \frac{\log(c'/c)}{dr} = -\frac{\log(c'/c)}{d\log[u'(c')/u(c)]} = \text{EIS}$

Inefficiencies in the Romer Model

- Monopolists generate deadweight losses to society:
  produce **less** than optimal
- Externalities also makes idea producers produce less

Role of Incentives

- But note that if $\pi = 0$, price of ideas is zero
- Therefore without monopolistic competitors in the intermediate good market, there would be no ideas
- It can be thought of as many small companies producing ideas they use in their products, without realizing the externalities

Role of Externalities

- The idea technology features **externalities**:

$$\dot{A} = \eta L_A A$$

- More ideas makes more ideas easier
- Monopolistic firms take A as given, only consider price of ideas and labor
- Research firms solve a single firm, single period profit maximization problem

Socially Optimal BGP

- Socially optimal level of int. good production is when $p_j = R$:

$$\begin{aligned} x_j &= (\alpha/R)^{\frac{1}{1-\alpha}} \cdot (1-\mu)L = K/A \\ \Rightarrow \quad k &= (\alpha/R)^{\frac{1}{1-\alpha}} \cdot (1-\mu)A \end{aligned}$$

- Same as if final good firm chooses x_j given price R;
  that is, f.o.c. for K in

$$\max_{K,L} \left\{ K^{\alpha}(AL_Y)^{1-\alpha} - RK - wL_Y \right\}$$

“BGP" Planner

- Given the BGP, a planner chooses A to solve

$$\max\left\{\int_0^\infty e^{-\rho t}\cdot\frac{c_t^{1-\gamma}}{1-\gamma}dt\right\}$$

with the resource constraints

$$c_t=\left[(1-\mu)A\right]^{1-\alpha}k^\alpha-(g+\delta)k,\qquad \dot{A}=\eta\mu LA$$

- Replace k to obtain

$$c_t=\underbrace{\left[(\alpha/R)^{\frac{\alpha}{1-\alpha}}-(g+\delta)(\alpha/R)^{\frac{1}{1-\alpha}}\right]}_{\equiv\kappa^{-1}}(1-\mu)A$$

$$\Rightarrow\quad \mu LA=L(A-\kappa c_t)$$

$$\Rightarrow\quad \dot{A}=\eta L(A-\kappa c_t)$$

Socially Optimal BGP Solution

- Optimality conditions are

$$\begin{cases} u'(c) = \lambda \eta L \kappa \\ \dot{\lambda}_t = -\lambda_t(\eta L - \rho) \end{cases} \quad \Rightarrow \quad \gamma g^P = \eta L - \rho$$

- Hence optimal research share must be

$$\mu^P = \frac{1 - \rho/\eta L}{\gamma}, \quad g^P = \frac{\eta L - \rho}{\gamma}$$

- Compare with equilibrium solution

$$\mu^* = \frac{\alpha - \rho/\eta L}{\alpha + \gamma}, \quad g^* = \frac{\alpha \eta L - \rho}{\alpha + \gamma}$$

1. Research share is higher: externality internalized, no monopoly
2. Capital lower and interest rate higher

Final Exam

- If you can solve the models in the slides and homeworks, it should be easy.
- Good luck!