Questions

1. *(Speed of Convergence in the Solow Model).* Recall the law of motion in the Solow model (in normalized units):

$$\dot{\hat{k}} = sf(\hat{k}) - (n + g + \delta)\hat{k}.$$

   (a) Let $g(\hat{k}) \equiv \dot{\hat{k}}$. Expand $g(\hat{k})$ around the steady state $\hat{k}^*$ by Taylor expansion. Solve the resulting ODE to show that the local dynamics of $\hat{k}$ around the steady state can be described by:

$$\hat{k}_t \equiv \hat{k}^* + (\hat{k}_0 - \hat{k}^*) \cdot e^{g'(\hat{k}^*)t}.$$

   (b) Check that the solution is stable, and define the speed of convergence as $|g'(\hat{k}^*)|$. How does the speed vary with s? With g? Try to give intuitive explanations as why the two effects differ. Why are such effects absent in the RCK model?

2. *(RCK Model in Equilibrium).* Now let's consider how the RCK model would look like in equilibrium. A representative household supplies labor inelastically and solves

$$\max_{c,a} U(c) = \int_0^\infty e^{-\rho t} u(c_t) dt$$
$$\text{s.t.} \quad \dot{A}_t = w_t L_t + r_t A_t - C_t,$$
$$A_0 > 0 \text{ given,}$$

where A_t is aggregate savings, L_t is aggregate labor and

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Remember that c_t is per capita consumption, while C_t is aggregate consumption.

A representative firm solves

$$\max_{K_t, L_t} \left\{ K_t^\alpha (Z_t L_t)^{1-\alpha} - R_t K_t - w_t L_t \right\}.$$

The population L_t grows at rate n, and technology Z_t at rate g. In equilibrium, the rental rate of capital satisfies $R_t = r_t + \delta$.

An equilibrium is defined as when both the representative household and firm solve their problems, and markets clear (goods, capital and labor markets clear; so $K_t = A_t$ and L_t is the same for both households and firms.

(a) Write down the current-value Hamiltonian of the representative household, $\mathcal{H}(\hat{c}_t, \hat{k}_t, \lambda_t)$, where λ_t is the costate. Normalize all variables into "effective" units: use lowercase variables to express per capita units, and "hats" for the effective units. Be clear which variables have to be normalized.

(b) Derive the optimality conditions for the representative household.

(c) Now, write down the first-order conditions for the representative firm. Transform the conditions into the effective units of K_t.

(d) Plug in the market clearing conditions $K_t = A_t$ into the household's optimality conditions. Verify that the resulting system of equations is the same as the version we solved in class, so the solution would be the same whether we consider the equilibrium or not.

3. *(Tax and Subsidies in the Romer Model).* Consider the Romer model we learned in class. Everything is the same, except that now there is a government that

- Subsidizes intermediate firms' cost of capital, so their profit maximization problem is

$$p_j(x_j)x_j - \frac{R}{1+\tau_s} \cdot x_j,$$

- Taxes firms to pay for labor, so the profit maximization problem of the final good firm and research firm are

$$\max_{L_Y, x_j} \left\{ L_Y^{1-\alpha} \int_0^Z x_j^\alpha dj - (1+\tau_w) w L_Y - \int_0^Z p_j x_j dj \right\}$$

and

$$\max_{L_Z} \left\{ P_Z \cdot \dot{Z} - (1+\tau_w) w L_Z \right\}$$

(a) Following the steps in class, derive expression for the f.o.c.'s of the final good firm, intermediate good firm, and research firm. Without the tax policies, this resulted in a research share of

$$\mu = 1 - r/\eta\alpha L.$$

How does the research share change once we include the policies?

(b) The BGP interest rate is still determined by the household's problem,

$$r = g + \hat{\rho} = \gamma g + \rho.$$

Derive expressions for (μ^*, g^*), the BGP research share and growth rate, with government policies.

(c) The socially optimal level of (μ^P, g^P) is

$$\mu^P = \frac{\alpha - \rho/\eta L}{\gamma}, \quad g^P = \frac{\eta L - \rho}{\gamma}.$$

Argue that the two government policies can be used to achieve the social optimum. Can the government achieve the social optimum with only one of the taxes? Why or why not?

last updated: December 31, 2017