Questions

1. *(Closed-form Solution in the Solow Model).* Recall the law of motion in the Solow model (in normalized units):

$$\dot{\hat{k}} = s\hat{k}^{\alpha} - (n + g + \delta)\hat{k}.$$

   (a) Let $v = \hat{k}^{1-\alpha}$. Obtain an expression for $\dot{v} \equiv dv_t/dt$, which is a non-homogeneous first-order differential equation, with solution

$$v(t) = \frac{s}{n + g + \delta} + Ce^{-(1-\alpha)(n+g+\delta)t},$$

   where C is an unknown constant.

   (b) Denote the value of $\hat{k}_t$ at time 0 as $\hat{k}_0$, and write down the expression for C. Given this, show that the full solution to the Solow model can be written as

$$\hat{k}(t) = \left[\frac{s}{n + g + \delta} + \left(\hat{k}_0^{1-\alpha} - \frac{s}{n + g + \delta}\right) e^{-(1-\alpha)(n+g+\delta)t}\right]^{\frac{1}{1-\alpha}}.$$

2. *(The Golden Rule).* Based on the model we learned in class with population growth n, technology growth g and depreciation rate δ.

   (a) Write down an expression for consumption per worker, and find the savings rate, $s = s^*$, that will maximize consumption per worker on the balanced growth path. This is the so-called "golden rule."

   (b) Suppose that we are at a BGP and that the savings rate is $s = s_0 < s^*$. Suddenly, s jumps to s^*—i.e., people start saving more. What happens to consumption per worker in the short run? What happens in the long run? Explain graphically and mathematically.

   (c) Given your answer above, why may the golden rule in fact not be golden unless you are already saving at $s = s^*$?

3. *(Wages and rental rates on a BGP).* Recall the standard Solow model with APF

$$F(K, ZL) = K^{\alpha}(ZL)^{1-\alpha}.$$

   (a) Assuming that the firm maximizes profits, solve for the optimal capital-labor ratio the firm would hire as a function of prices R and w. In other words, find a function $k(R, w)$ that tells you how many units of capital the firm would use per number of people hired.

(b) The firm doesn't care whether it's on a BGP or not, but we do. Now suppose we are on the BGP in the original Solow model with the usual parameters, n, g and δ. Since we know exactly what K and L are on the BGP, and we know the capital and labor demands, we can solve for the BGP levels of R and w. What are they? Are they constant? Why or why not?

(c) Now instead of the Solow model, assume we are in a world where the APF is

$$F(K, ZL) = K^{\alpha}(hZL)^{1-\alpha}$$

and human capital h is determined by $h = e^{\psi u}$. First obtain an expression for the BGP wage (be careful to make sure that wage w is what is paid to a *person*, not some efficient unit of worker). Then show that a marginal increase in u will increase wages by $\psi \times 100$ percent along the BGP.

(d) Now assume we are in a world where

$$F(K, ZL) = K^{\alpha}(Z(1-u)hL)^{1-\alpha}$$
$$\frac{dh(t)}{dt} = \psi u h(t)$$

and again find the BGP wage. Show that a marginal increase in u will increase the *growth rate* of wages by $\psi \times 100$ percent. How would you explain the different interpretations of ψ in questions (c) and (d)?

4. *(Mankiw, Romer, and Weil, 1992).* A seminal paper in growth is Mankiw et al. (1992). They assume identical-looking laws of motion for both physical and human capital:

$$Y = K^{\alpha}H^{\beta}(ZL)^{1-\alpha-\beta} \quad (1)$$
$$\dot{K} = s_K Y - \delta_K K$$
$$\dot{H} = s_H Y - \delta_H H$$

where the H subscripts is to differentiate the human capital parameters from their physical capital counterparts.

(a) Solve for the BGP, and discuss how it differs from what we learned in class.

(b) Using your solution for the BGP, construct an empirically testable equation. Given this equation, list at least one testable implication.

(c) Now forget about the BGP, and using equation (1) only, derive a testable equation along a time series. Again, list at least one testable implication.

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References

Mankiw, N. G., D. Romer, and D. N. Weil (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics 107*(2), pp. 407–437.